1. The astrophysical r-process is thought to occur in an environment with such a high neutron densities and temperatures that neutron captures and photodissociation reactions operate in a time scale much shorter than beta decay. Under these conditions, an equilibrium between neutron captures and photodissociation reactions develops. Show that this equilibrium implies that the nuclei that participate in the r-process have a constant neutron density that depends on the temperature and neutron density of the environment, $S_n^0(T, n_n)$.

   (a) Assuming a temperature of 1.3 GK and a neutron density of $n_n = 5\times10^{21}$ cm^{-3} determine the r-process path: the set of nuclei that have a neutron separation energy such as $S_n(Z, A) \gtrsim S_n^0(T, n_n)$. Use the neutron separation energies from ref. [1].

   (b) If the astrophysical duration of the r-process is much larger than the sum of beta-decays for the nuclei that constitute the r-process path, the r-process reaches an equilibrium that is denoted as beta-flow approximation in which the abundance of the nuclei in the r-process path is proportional to their beta-decay half-live. Determine the abundances of the r-process nuclei for the conditions above using the beta decay half-lives of ref. [2]. Where are located the nuclei with largest abundances? Can you estimate the duration of the r-process?

2. Neutrinos from Supernova 1987A, which was located at a distance from Earth of about 50 kpc, were observed at KAMIOKANDE-2 (Cerenkov detector, 2140 tons of H_2O). The electron antineutrinos, $\bar{\nu}_e$, are detected via inverse β-decay

$$\bar{\nu}_e + p \longrightarrow e^+ + n, \tag{1}$$

   with a cross section $\sigma_{\bar{\nu}_e p} \simeq 10^{-41}$ cm^2. The mean energy of the antineutrinos was $\langle E_{\bar{\nu}_e} \rangle \simeq 15$ MeV. Kamiokande detected 12 events during a time interval of 10 seconds.

   (a) Estimate the total energy emitted in neutrinos assuming that equal number of neutrinos and antineutrinos are emitted in each flavor.

   (b) Determine the neutrino luminosity for a particular flavor.

   (c) Determine the total number of neutrinos emitted.

3. The protoneutron star that forms after a core-collapse supernova explosion cools by neutrino emission. The typical luminosity has been estimated in the previous exercise. Assume a protoneutron star 20 km radius and 1.5 $M_\odot$.

   (a) Determine the surface temperature assuming that the star emits neutrinos like a blackbody, i.e. apply the Stefan-Boltzmann law. What will be the composition at the surface? (heavy nuclei or nucleons)

   (b) Determine the gravitational binding energy of a nucleon, a proton or a neutron, located at the surface.

(c) This nucleon will be subject to an intense flux of neutrinos that can be absorbed by the reactions: $\bar{\nu}_e + p \to e^+ + n$ and $\nu_e + n \to e^- + p$. Assuming the same average energy and cross section for the neutrinos than in the previous exercise, determine the energy transferred by each neutrino or antineutrino absorption. How many neutrino absorptions are necessary to unbound the nucleon?

(d) Assuming that one neutrino of each 10^6 is absorbed determine the total amount of matter ejected.

4. During the collapse of a massive star neutrinos become trapped once the core reaches densities between 10^{11} and 10^{12} g cm^{-3}. The main mechanism responsible for neutrino trapping is neutrino elastic scattering with nuclei with neutron number N. The cross section for a neutrino energy E is given by:

$$\sigma(E) = \sigma_0 E^2 N^2, \quad \sigma_0 = 4.2 \times 10^{-45} \text{ cm}^2 \text{ MeV}^{-2} \tag{2}$$

The mean free path is related to the cross section, $l = 1/(n\sigma)$ with n the number density of nuclei. Determine the mean free path and the time for neutrinos to diffuse out of the central core region of radius 30 km for densities 10^{11} and 10^{12} g cm^{-3}. Compare with the collapse time scale at each density given by $t_{coll} = 1/(G\rho)^{1/2}$. (This is the free fall time scale). Assume that the composition is given by nuclei with $A = 110$ and $Z = 40$.

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