

The Analysis of Matter in QCD

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Lecture 0: The Thermodynamics of Quarks and Gluons

Lecture 1: The States of Matter in QCD

Lecture 2: The Spectral Analysis of the QGP

Lecture 3: The Origin of Thermal Hadron Production

The Thermodynamics of Quarks and Gluons

The Fundamental Problems of Physics

constituents

quarks
leptons
gluons, photons
vector bosons (Z , W^\pm)
Higgs

forces

strong
e-m
weak
gravitation
unification, TOE

elementary interactions



complex systems, critical behaviour

states of matter

solid, liquid, gas
glass, gelatine
insulator, conductor
superconductor, ferromagnet
fluid, superfluid

transitions

thermal phase transitions
percolation transitions
scaling and renormalization
critical exponents
universality classes

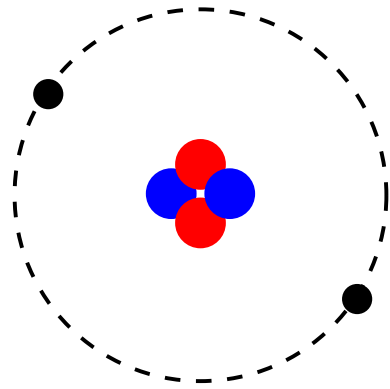
Complex Systems \Rightarrow **New Direction** in Physics

- Given constituents and dynamics of elementary systems, what is the behaviour of complex systems?
- What are the possible states of matter and how can they be specified?
- How do transitions from one state of matter to another occur?
- Is there a general pattern of critical phenomena, independent of specific dynamics?
- Conceptually new physics: renormalization, self-similarity, self-organization, emergence, sand piles, swarm intelligence, ...

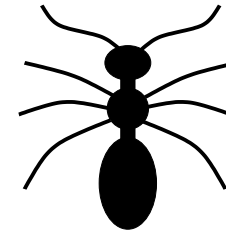
NB: new directions not only in physics

Knowing all there is to know about

the helium atom

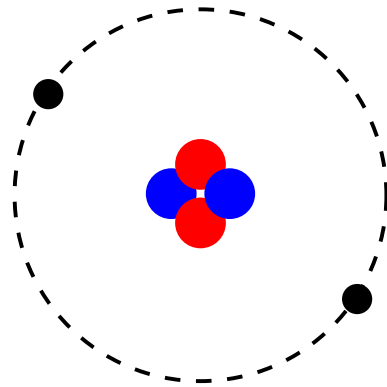


the ant

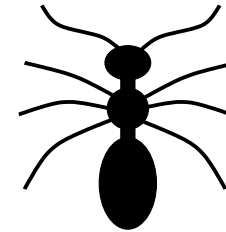


Knowing all there is to know about

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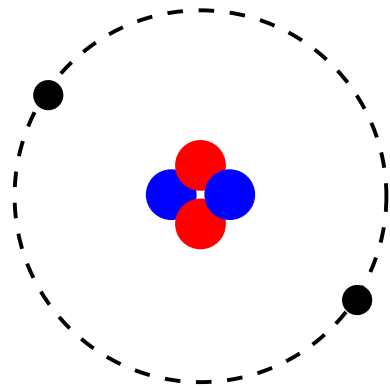
tells you nothing about the behaviour of

liquid helium

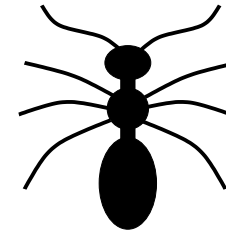
a colony of ants

Knowing all there is to know about

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tells you nothing about the behaviour of

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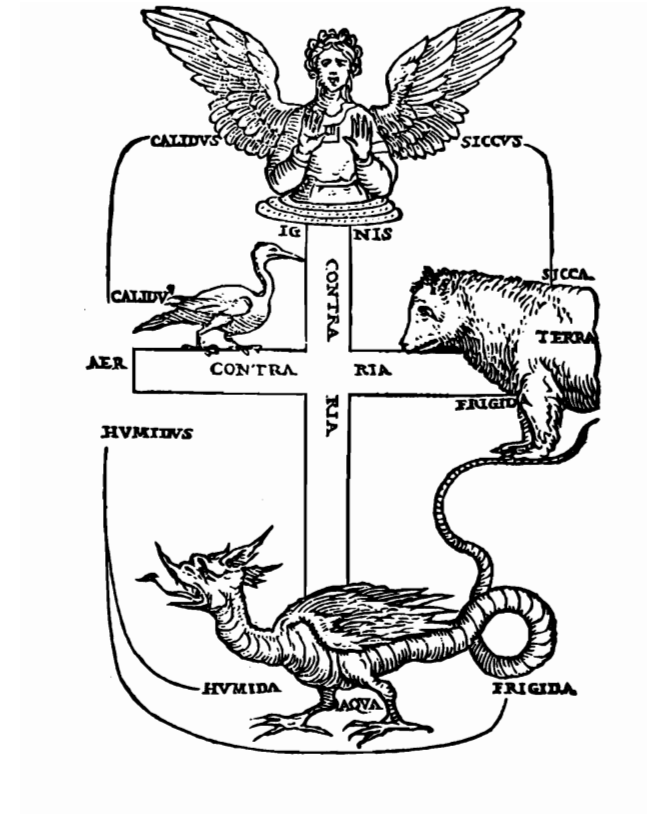
a colony of ants

⇒ even a fully unified fundamental theory does not solve the issue of complex systems: in physics, what are the states of matter?

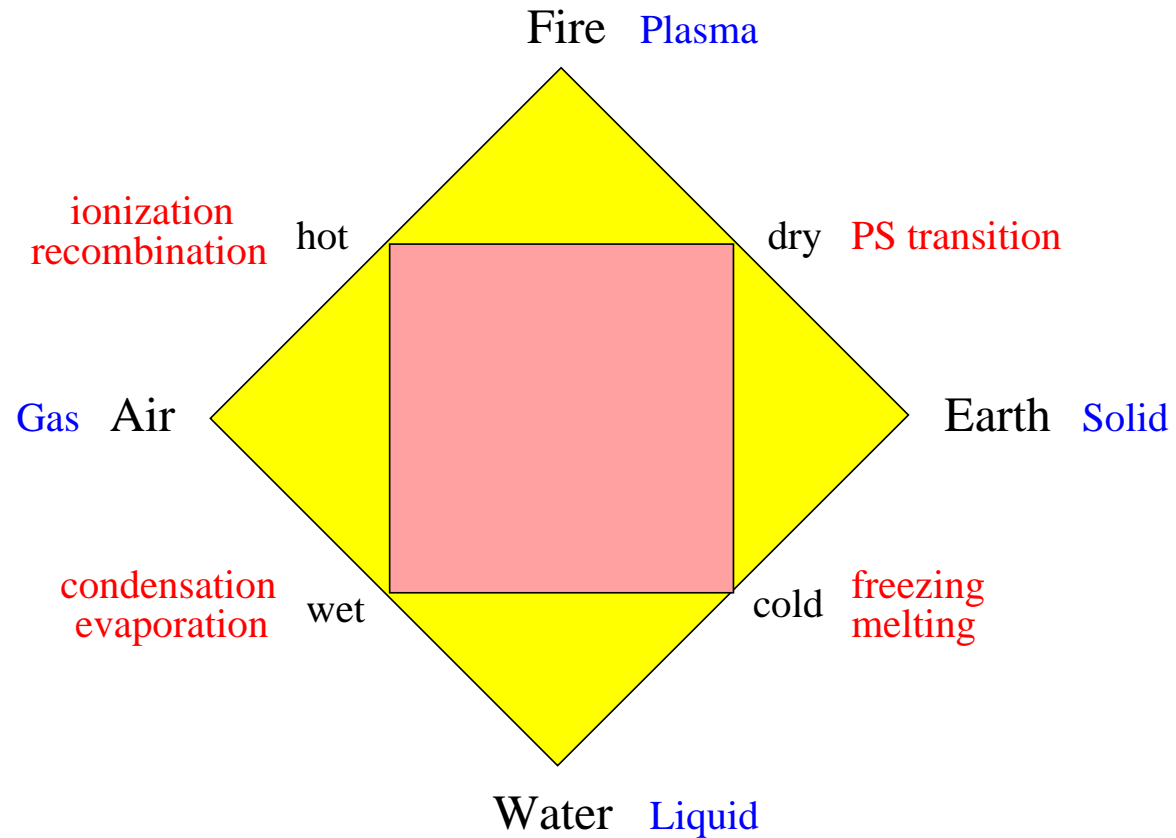
The States of Matter 500 B. C. - Experiment



The States of Matter 500 B. C. - Theory



The States of Matter 500 B. C. - Theory



Contents

1. States of Strongly Interacting Matter
2. From Hadrons to Quarks and Gluons
3. Finite Temperature Lattice QCD
4. The Nature of the Transition
5. Probing the Quark-Gluon Plasma

Summary

1. States of Strongly Interacting Matter

What happens to strongly interacting matter at high temperature and/or density?

- hadrons have intrinsic size $r_h \simeq 1$ fm, need $V_h \simeq (4\pi/3)r_h^3$ to exist

⇒ limiting density of hadronic matter

$$n_c = 1/V_h \simeq 1.5 n_0 \quad [\text{Pomeranchuk 1951}]$$

- resonances → exponential hadron spectrum $\rho(m) \sim \exp(bm)$

– statistical bootstrap model [Hagedorn 1968]

– dual resonance model

[Fubini & Veneziano 1969; Bardakçi & Mandelstam 1969]

⇒ limiting temperature of hadronic matter

$$T_c = 1/b \simeq 150 - 200 \text{ MeV}$$

⇒ what lies beyond n_c, T_c ? ⇐

- quark liberation

hadronic matter: colorless constituents of hadronic dimension



quark-gluon plasma: pointlike colored constituents

⇒ deconfinement: insulator-conductor transition in QCD

- quark mass shift

at $T = 0$, quarks ‘dress’ with gluons → constituent quarks

bare quark mass $m_q \sim 0$ → constituent quark mass $M_q \sim 300$ MeV

in hot medium, dressing ‘melts’ $M_q \rightarrow 0$

for $m_q = 0$, \mathcal{L}_{QCD} has chiral symmetry

$M_q \neq 0$ → spontaneous chiral symmetry breaking

$M_q \rightarrow 0$ ⇒ chiral symmetry restoration

NB: **first** deconfinement, **then** chiral symmetry restoration

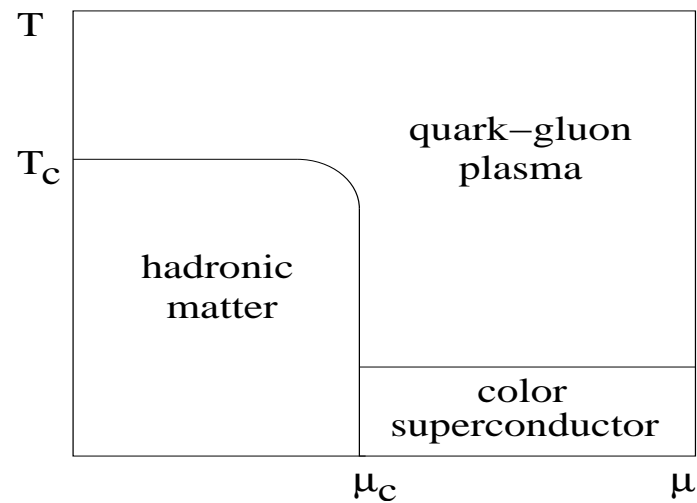
- diquark matter

deconfined quarks \sim attractive interaction

can form colored bosonic ‘diquark’ pairs (QCD’s Cooper pairs)

form condensate \Rightarrow color superconductor

- expected phase diagram of QCD:



baryochemical potential $\mu \sim$ baryon density.

2. From Hadrons to Quarks and Gluons

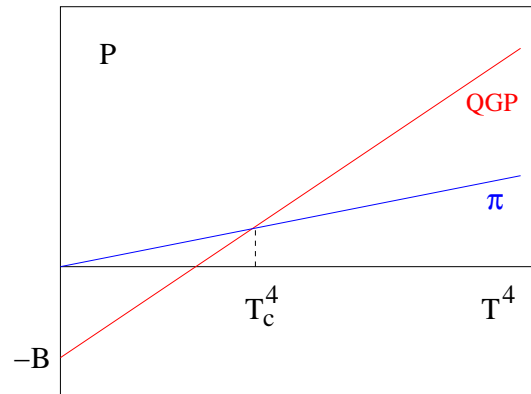
simplest confined matter: ideal pion gas $P_\pi = \frac{\pi^2}{90} \cdot 3 T^4 \simeq \frac{1}{3} T^4$

simplest deconfined matter: ideal quark-gluon plasma

$$P_{QGP} = \frac{\pi^2}{90} \{ 2 \times 8 + \frac{7}{8} [2 \times 2 \times 2 \times 3] \} T^4 - B \simeq 4 T^4 - B$$

with bag pressure B for outside/inside vacuum

\Rightarrow compare $P_\pi(T)$ and $P_{QGP}(T)$ vs. T



phase transition from hadronic matter at low T to QGP at high T

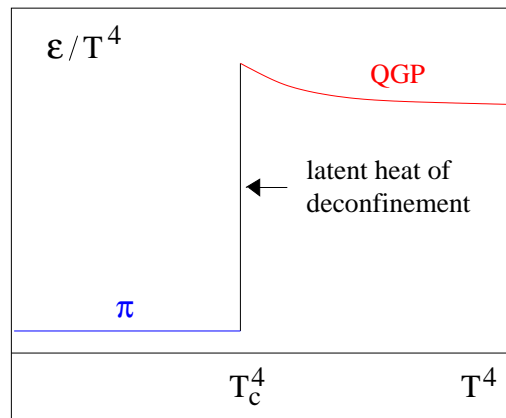
critical temperature:

$$P_\pi = P_{QGP} \rightarrow T_c^4 \simeq 0.3 B \simeq 150 \text{ MeV}$$

with $B^{1/4} \simeq 200 \text{ MeV}$ from quarkonium spectroscopy

corresponding energy densities

$$\epsilon_\pi \simeq T^4 \rightarrow \epsilon_{QGP} \simeq 12 T^4 + B$$



at T_c , energy density changes abruptly by latent heat of deconfinement

compare energy density and pressure:

ideal gas $\epsilon = 3P$

here we obtain

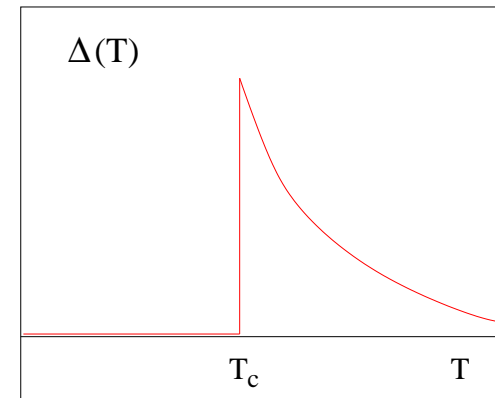
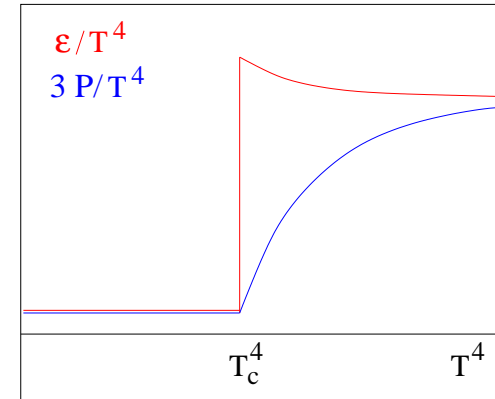
and the interaction measure

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = \frac{4B}{T^4}$$

shows that for $T_c \leq T < 2 - 3 T_c$

the QGP is strongly interacting

so far, simplistic model; real world?



3. Finite Temperature Lattice QCD

given QCD as **dynamics** input, calculate resulting **thermodynamics**,
based on **QCD partition function**

⇒ **lattice regularization**

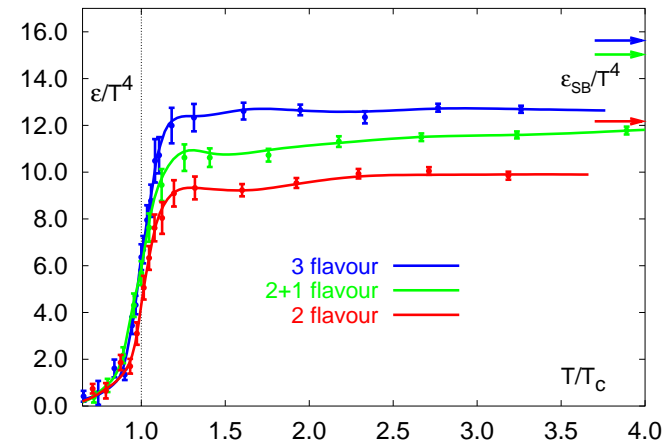
- energy density

⇒ **latent heat of deconfinement**

For $N_f = 2, 2 + 1$:

$$T_c \simeq 175 \text{ MeV}$$

$$\epsilon(T_c) \simeq 0.5 - 1.0 \text{ GeV}/\text{fm}^3$$



explicit relation to deconfinement, chiral symmetry restoration?

⇒ order parameters

- deconfinement

$$\Rightarrow m_q \rightarrow \infty$$

Polyakov loop $L(T) \sim \exp\{-F_{Q\bar{Q}}/T\}$

$F_{Q\bar{Q}}$: free energy of $Q\bar{Q}$ pair for $r \rightarrow \infty$

$$L(T) \begin{cases} = 0 & T < T_L \text{ confinement} \\ \neq 0 & T > T_L \text{ deconfinement} \end{cases}$$

variation defines deconfinement temperature T_L

- chiral symmetry restoration

$$\Rightarrow m_q \rightarrow 0$$

chiral condensate $\chi(T) \equiv \langle \bar{\psi}\psi \rangle \sim M_q$

measures dynamically generated ('constituent') quark mass

$$\chi(T) \begin{cases} \neq 0 & T < T_\chi \text{ chiral symmetry broken} \\ = 0 & T > T_\chi \text{ chiral symmetry restored} \end{cases}$$

variation defines chiral symmetry temperature T_χ

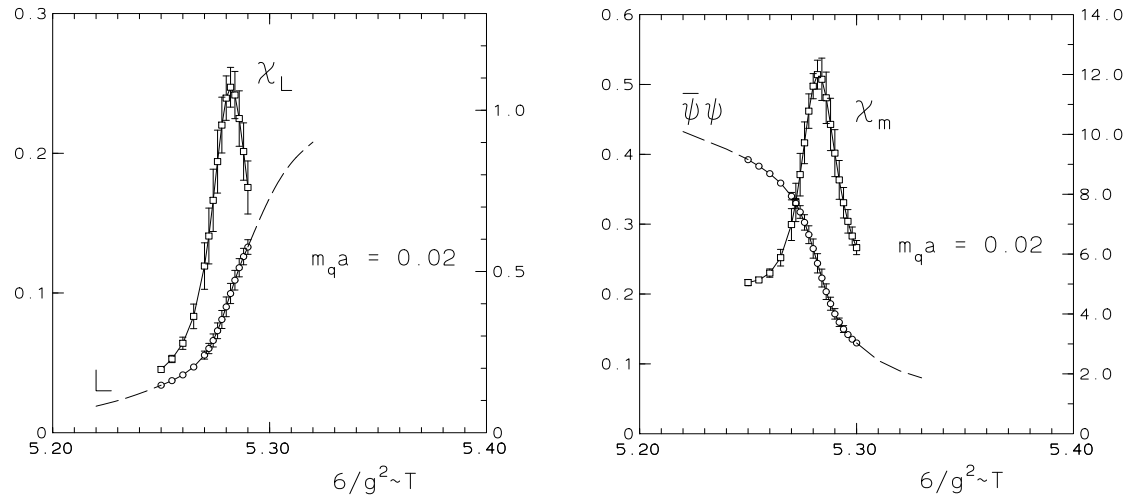
- how are T_L and T_χ related?

$SU(N)$ gauge theory: \sim spontaneous Z_N breaking at T_L

QCD, chiral limit: \sim explicit Z_N breaking by $\chi(T) \rightarrow 0$ at T_χ

chiral symmetry restoration \Rightarrow deconfinement

lattice results



Polyakov loop & chiral condensate vs. temperature

at $\mu = 0$, \exists one transition hadronic matter \rightarrow QGP

for $N_f = 2$, $m_q \rightarrow 0$ at $T_c = T_L = T_\chi \simeq 175$ MeV

- interaction measure peaks above T_c

at inflection point

$$(\partial\Delta/\partial T) \sim (\partial\epsilon/\partial T)$$

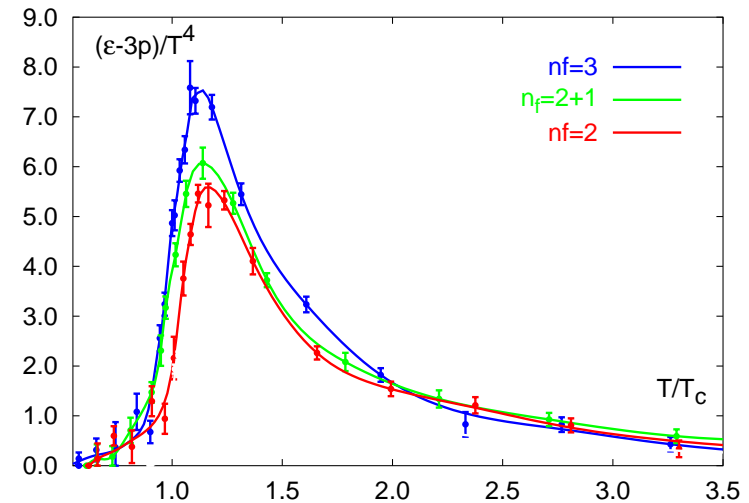
$$\sim C_v(T) \rightarrow \infty$$

(for continuous transition)

two regimes of QGP:

strongly interacting QGP (sQGP) for $T_c \leq T \leq (2 - 2.5)T_c$

weakly interacting QGP (wQGP) for $T \geq (2 - 2.5)T_c$



Finite temperature lattice QCD shows:

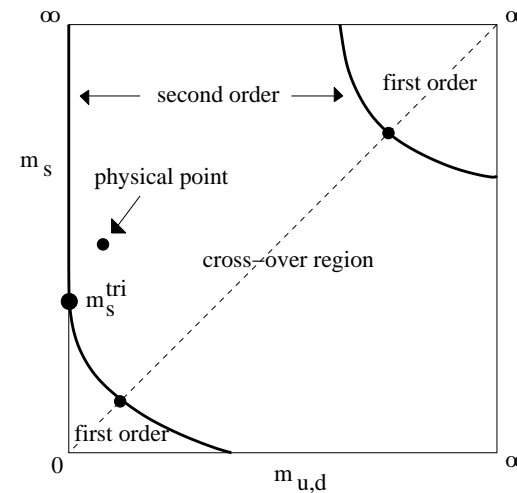
- \exists transition at $T \sim 0.175 \pm ? \text{ GeV}$, where deconfinement & chiral symmetry restoration coincide
- at transition, ϵ increases suddenly by latent heat of deconfinement

4. The Nature of the Transition

- for $m_q \rightarrow \infty$ (pure gauge theory)
spontaneous Z_N breaking \rightarrow **deconfinement transition**
- for $m_q \rightarrow 0$, spontaneous chiral symmetry breaking \rightarrow **chiral transition**
- for finite quark masses, no spontaneous symmetry breaking or restoration, hence in general no singular behaviour
- both $L(T)$ and $\chi(T)$ vary sharply for all m_q , define common transition point T_c
- what kind of transition?

depends on N_f and m_q :

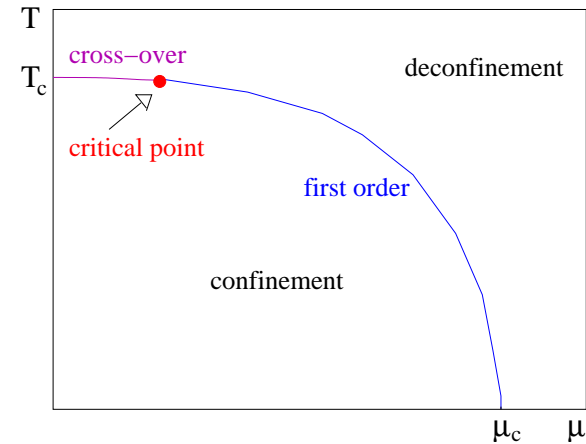
continuous, first order
“rapid” cross-over



- non-zero net baryon density

$$(\mu \neq 0, N_b > N_{\bar{b}}, N_f = 2 + 1)$$

computer algorithms break down:
reweighting, analytic continuation,
power series...; expect:



critical point in $T-\mu$ plane depends on position of physical point
in $m_s - m_{u,d}$ plane

- cross-over region (the real world): enigmatic
 - no thermal singularity, no thermal phase transition
 - so what does it mean: new state of matter?
 - observables change rapidly
 - clear transition in entire region: why?
 - what is the transition mechanism?

Small excursion into new lands: geometric critical behaviour

there is more on earth than traditional phase transitions such as freezing water or magnetization

what about making pudding, boiling an egg, ... ?

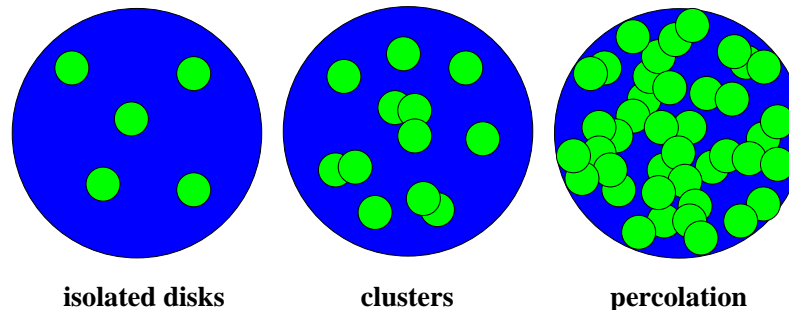
(sol-gel transitions)

⇒ cluster formation & percolation

divergence of **geometric** observables (cluster size,...) rather than **thermodynamic** observables (specific heat,...)

two-dimensional disk
percolation:

lilies on a pond



distribute small disks of area a randomly on large area $A \gg a$, with overlap allowed: when can an ant walk across?

for constituents with intrinsic scale,

⇒ formation of infinite connected clusters or networks

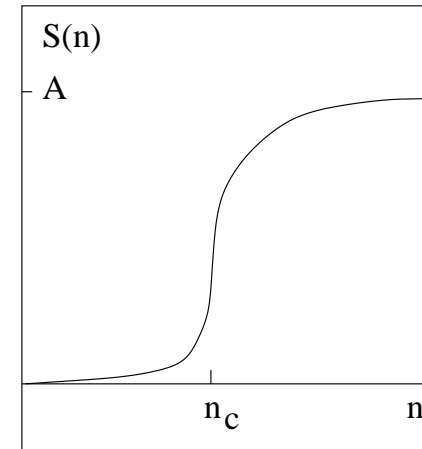
average cluster size $S(n)$ increases
with increasing density $n = N/A$

suddenly, for $n \rightarrow n_c$, $S(n)$ becomes
large enough to span the pond: $S \sim A$

for $N \rightarrow \infty$, $A \rightarrow \infty$:

$S(n_c)$ and $(dS(n)/dn)_{n=n_c}$ diverge:

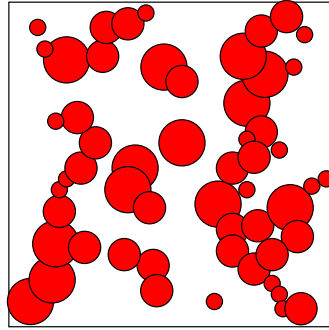
⇒ percolation



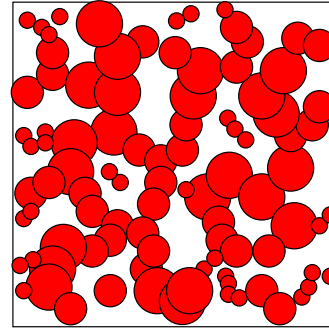
2d (disks): $n_c \simeq 1.13/\pi r^2$, 68 % of space covered, 32 % empty
when an ant can cross, a ship cannot, and vice versa: 2d effect

3d (spheres): $n_c \simeq 0.34/(4\pi/3) r^3$, 29 % of space covered, 71 %
empty; both cluster and empty space connected

$\bar{n}_c \simeq 1.24/(4\pi/3) r^3$, 71 % of space covered, 29 %
empty; connected vacuum disappears



onset of cluster



end of vacuum

percolation

apply to hadrons: $r = r_h \simeq 0.8 \text{ fm}$

$\Rightarrow n_c \simeq 0.16 \text{ fm}^{-3} \sim$ normal nuclear matter

$\Rightarrow \bar{n}_c \simeq 0.56 \text{ fm}^{-3}$ consider ideal gas of hadronic resonances

$n_{\text{res}}(T_c) = \bar{n}_c \Rightarrow T_c \simeq 170 \text{ MeV} \sim$ deconfinement

can use geometric critical behaviour to define the states of strongly interacting matter

thermodynamic critical behaviour \subset geometric critical behaviour

5. Probing the Quark-Gluon Plasma

At high temperatures and/or densities, strongly interacting matter becomes a QGP;

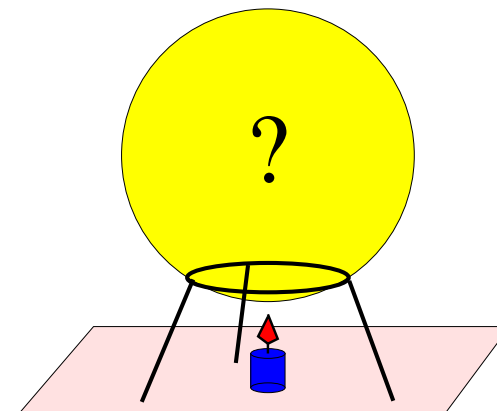
how can we probe its properties and its behaviour as function of temperature and density?

A. Einstein: make things as simple as possible, but not simpler.

Given a volume of strongly interacting matter and an energy source, how can we determine its state at different temperatures?

NB:

equilibrium thermodynamics, no collision effects, time dependence, equilibration, etc.



- Possible probes:
- hadron radiation
 - electromagnetic radiation
 - dissociation of quarkonium states
 - energy loss of parton beams

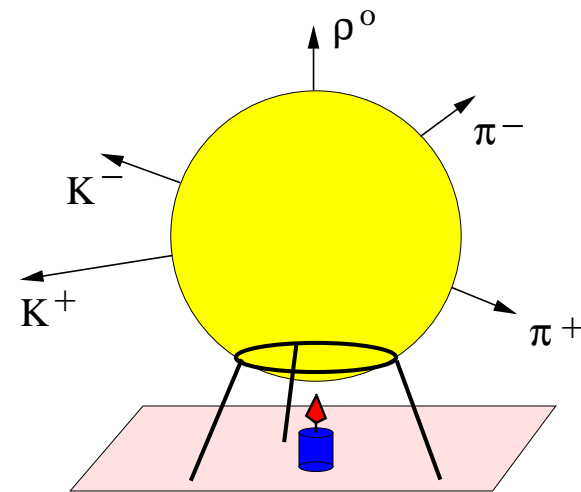
Here, just a brief first look....

The medium is **hotter** than its environment (vacuum) and hence emits

- Hadron Radiation

emission of light hadrons
(made of u, d, s quarks)
scale $\sim 1 \text{ fm} \simeq 1/(200 \text{ MeV})$

cannot exist in hot interior
emission at surface of $T \simeq T_c$
information about hadronization stage



\Rightarrow same relative abundances for different initial energy densities

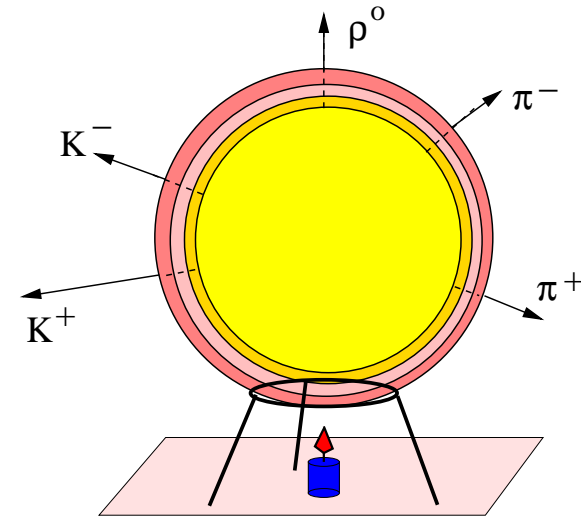
Hadron emission: no information about pre-hadronic medium

BUT:

if medium not contained,
it can expand freely

⇒ Hydrodynamic Flow

- “radial flow”: boosts hadron momenta



non-spherical initial state (peripheral collisions)

⇒ spatially different pressure gradients

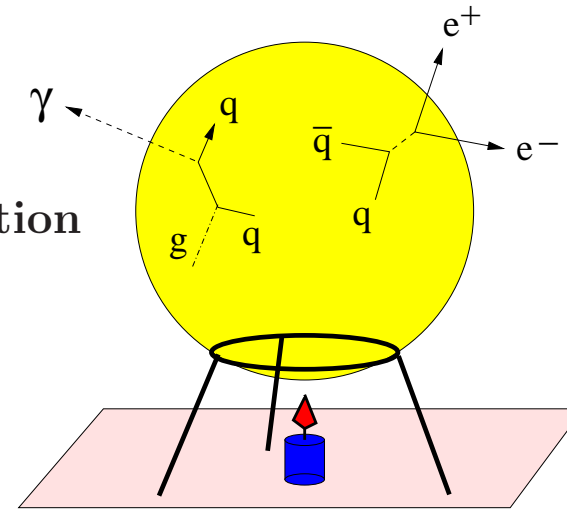
- “directed” or “elliptic” flow, boost depends on spatial directions

both forms of flow depend on conditions of medium in all stages and hence can (in principle) also provide information about hot QGP

In the interior of the medium, quark-gluon interactions or quark-antiquark annihilation leads to

- Electromagnetic Radiation

produced photons and dileptons
leave medium without further interaction
provide information about the medium
at the time of their production:
probe of hot QGP



problem:

they can be formed anywhere & at any time
even at the surface or by the emitted hadrons
task: identify hot thermal radiation

hadronic and e-m radiation: emitted by the medium itself
provide information about the medium at the time of production

other possibility: “outside” probes

- Quarkonium Suppression

quarkonia: bound states of heavy quarks ($c\bar{c}$, $b\bar{b}$)

smaller than usual hadrons ($r_Q \ll r_h \simeq 1$ fm),
binding energies 0.5 – 1.0 GeV

\Rightarrow can survive in QGP
in some temperature range $T > T_c$

Example: charmonium states

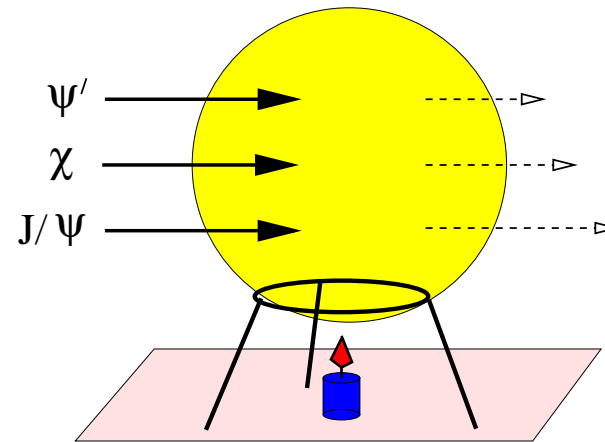
$J/\psi(1S) - r_{J/\psi} \simeq 0.2$ fm

$\chi_c(1P) - r_\chi \simeq 0.3$ fm

$\psi'(2S) - r_{\psi'} \simeq 0.4$ fm

different charmonia “melt”
in QGP at different temperatures
(confirmed by lattice QCD)

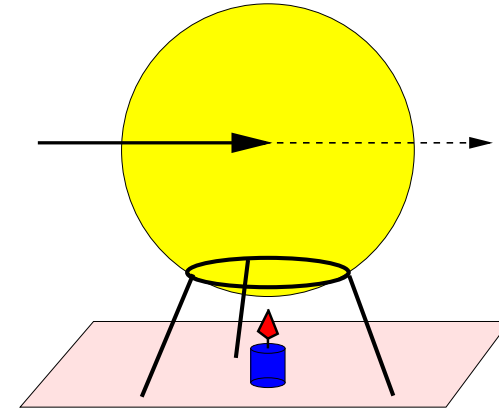
\Rightarrow “sequential charmonium melting pattern” as quantitatively
predicted property of QGP



- Jet Quenching

shoot an energetic parton beam
(quarks or gluons) into QGP,
measure energy of outgoing beam

attenuation (“quenching”)
determined by density of medium
increases with temperature



NB: how to get “external” probes in nuclear collision experiments?

- Hard Probes:

quarkonia, open charm/beauty, jets, energetic photons & dileptons
– formed very early in the collision, are present when QGP appears
– can be predicted (to large extent) by perturbative QCD
– can be “gauged” in pp and pA collisions

Summary

In strong interaction thermodynamics \exists a well-defined transition at which

- **deconfinement** sets in
- **chiral symmetry** is restored
- **latent heat** increases energy density.
- The transition temperature is $T_c \simeq 160 - 190 \text{ MeV}$.

For $T > T_c$, the state of matter is a plasma of deconfined quarks and gluons which can be probed by

- **electromagnetic radiation**
- **quarkonium spectra**
- **jet quenching**
- **flow aspects** of hadron radiation.