

The Analysis of Matter in QCD

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Lecture 0: The Thermodynamics of Quarks and Gluons

Lecture 1: The States of Matter in QCD

Lecture 2: The Spectral Analysis of the QGP

Lecture 3: The Origin of Thermal Hadron Production

**The Spectral Analysis
of the QGP**

Contents

1. Quarkonia
2. Thermodynamics of Quarkonium Dissociation
3. Dynamics of Quarkonium Dissociation
4. Quarkonium Production in Nuclear Collisions
5. Conclusions

1. Quarkonia

heavy quark ($Q\bar{Q} = c\bar{c}, b\bar{b}$) bound states **stable** under strong decay

heavy: charm ($m_c \simeq 1.3 \text{ GeV}$) or beauty ($m_b \simeq 4.7 \text{ GeV}$)

stable: $M_{c\bar{c}} \leq 2 M_D$ and $M_{b\bar{b}} \leq 2 M_B$

$M_{D,B}$ open charm/beauty mesons ($c\bar{u}$)/($b\bar{u}$)...

heavy quarks:

\Rightarrow quarkonium spectroscopy via non-relativistic potential theory

confining (“Cornell”) potential for $Q\bar{Q}$ at separation distance r ,

$$V(r) = \sigma r - \frac{\alpha}{r}$$

with string tension $\sigma \simeq 0.2 \text{ GeV}^2$, gauge coupling $\alpha \simeq \pi/12$

Schrödinger equation

$$\left\{ 2m_c - \frac{1}{m_c} \nabla^2 + V(r) \right\} \Phi_i(r) = M_i \Phi_i(r)$$

determines bound state masses M_i and wave functions $\Phi_i(r)$

wave functions \rightarrow average radii $\langle r_i^2 \rangle = \int d^3r r^2 |\Phi_i(r)|^2$

binding energies $\Delta E \equiv 2 M_{D,B} - M_i$

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
ΔE [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
ΔM [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
radius [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

- N-R potential theory provides excellent account of quarkonium spectroscopy

Ground states $J/\psi, \Upsilon$

tightly bound $2M_{D,B} - M_{J/\psi,\Upsilon} \gg \Lambda_{QCD}$

small size $r_{J/\psi,\Upsilon} \ll r_h$

How can they be dissociated?

2. Quarkonium Dissociation in QCD Thermodynamics

2.1 Heavy Quark Binding in Media

What happens if we separate Q and \bar{Q} ?

- in vacuum

confining string energy

$$F(r) \sim \sigma r$$

string breaking for $F(r) \geq F_0$

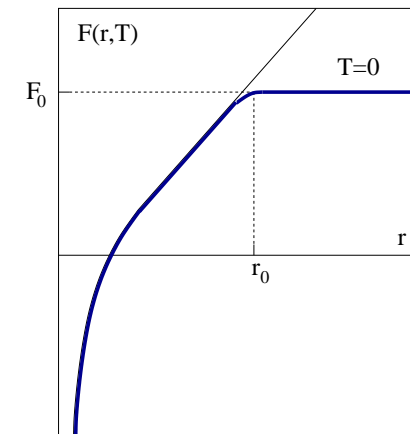
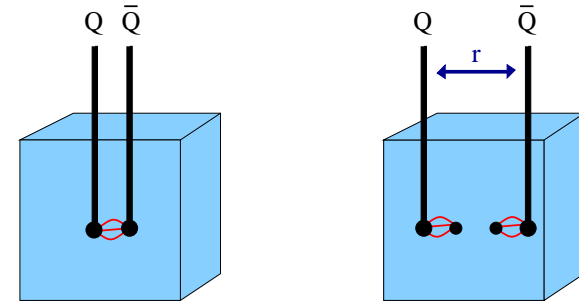
\Rightarrow light-heavy mesons $(Q\bar{q}), (\bar{Q}q)$

String breaking energy for charm

$$F_0 = 2(M_D - m_c) \simeq 1.2 \text{ GeV}$$

and for bottom

$$F_0 = 2(M_B - m_b) \simeq 1.2 \text{ GeV}$$



String breaking occurs when charges are separated by

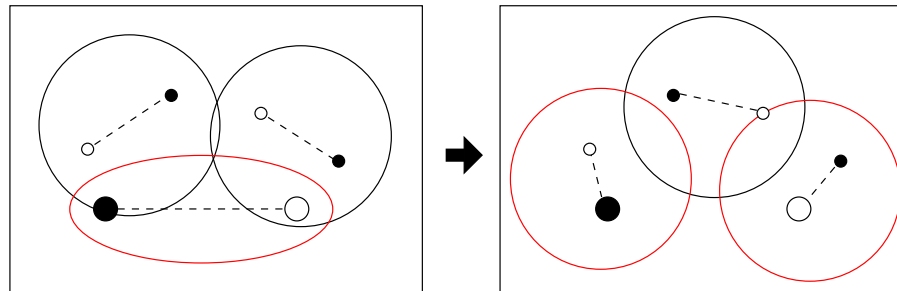
$$r_0 \simeq 1.2 \text{ GeV}/\sigma \simeq 1.5 \text{ fm}$$

property of “vacuum as medium at $T = 0$ ”

- in medium, $0 < T < T_c$

medium now contains normal hadrons (mesons)

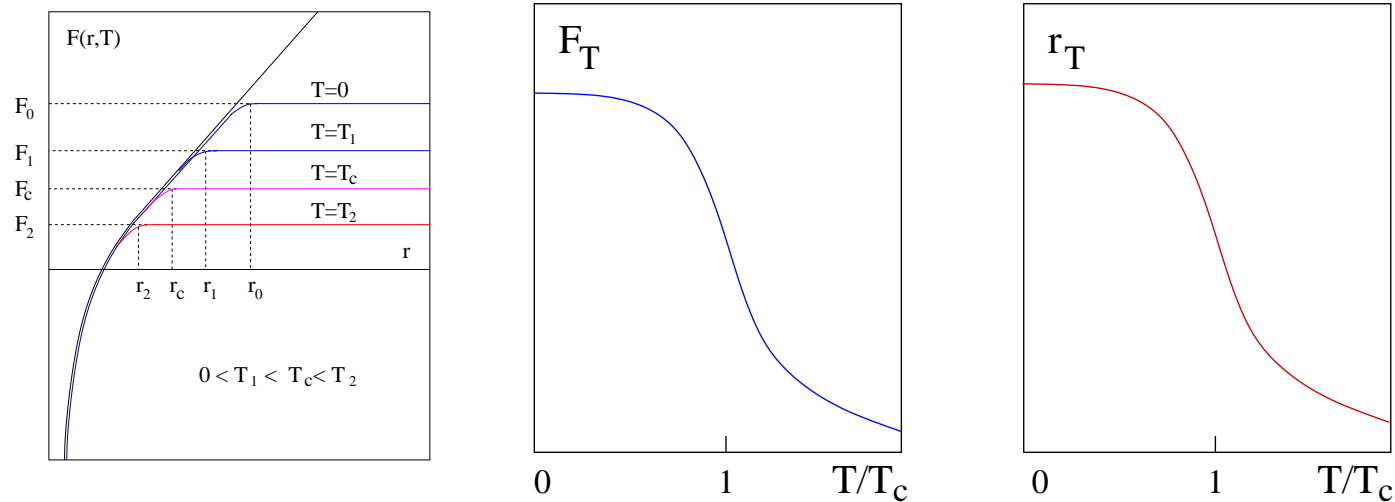
overlap \Rightarrow dissociation via **quark recombination**



increasing T increases hadron density, lowers dissociation energy,

shortens dissociation separation \Rightarrow effective screening

Near deconfinement point $T = T_c$, strong density increase and consequences



- in medium, $T > T_c$

medium now consists of unbound colour charges

polarization around Q and \bar{Q} : \exists **colour screening**

\Rightarrow screening radius $r_D(T)$ determines range of force

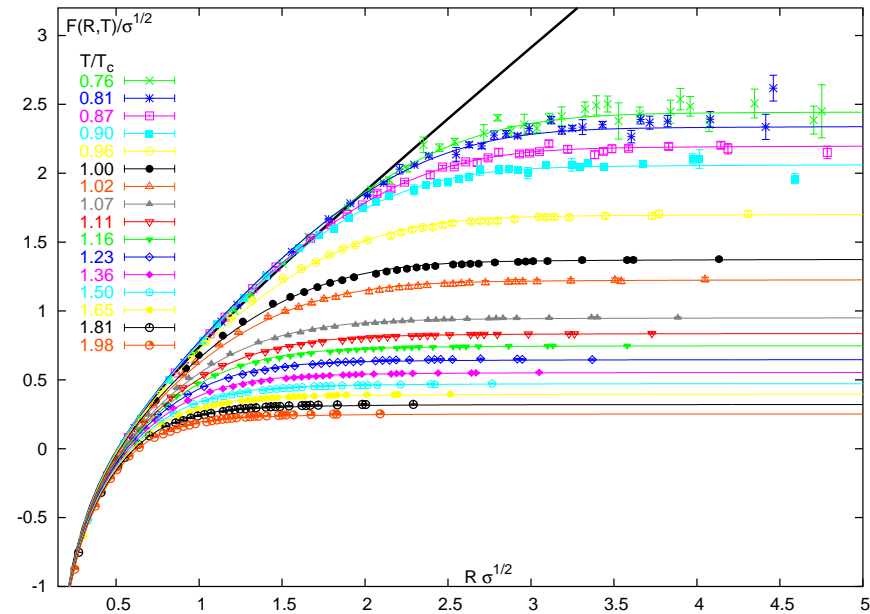
dissociation distance r_T and energy F_T decrease further with T

Conceptually clear: \exists three types of separation mechanisms

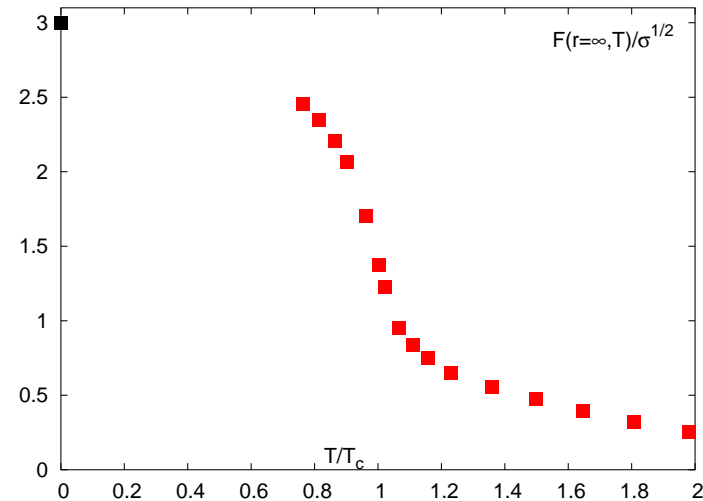
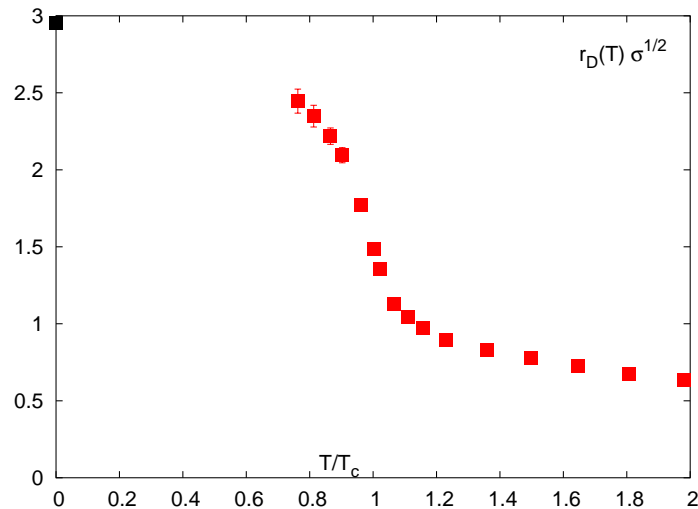
- $T = 0$: string breaking
- $0 < T < T_c$: quark recombination \sim effective screening
- $T_c < T$: colour screening

What is the quantitative effect of these mechanisms?

$N_f = 2$ lattice QCD:



Breaking point specifies force range,
large distance behaviour specifies maximum binding energy



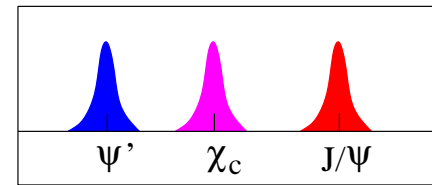
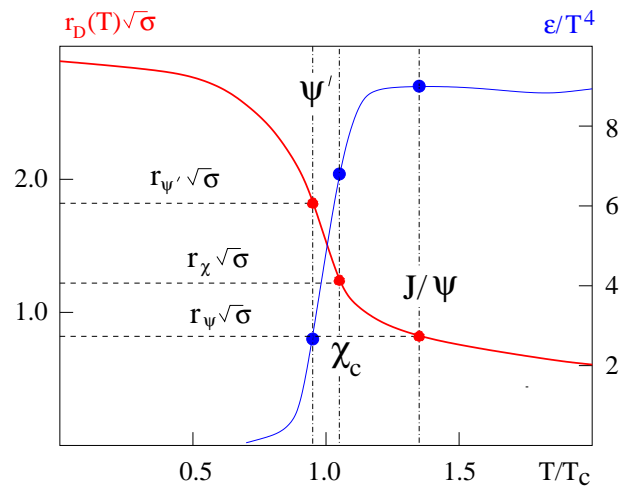
strong density increase near T_c causes strong decrease in both

What happens when **force range** < **quarkonium radius**?

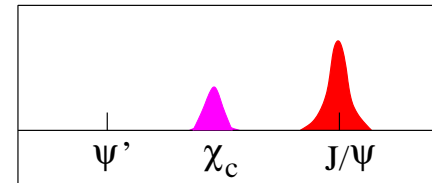
Q and \bar{Q} inside quarkonium cannot “see” each other any more:

\Rightarrow quarkonium **dissociates**

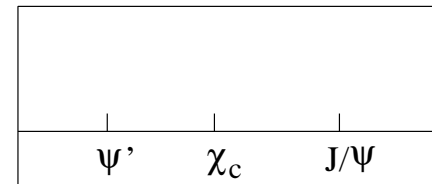
⇒ dissociation points of quarkonia determine temperature, energy density of medium



$T < T_c$



$T \sim 1.1 T_c$



$T \gg T_c$

How can one calculate quarkonium dissociation points?

Two possibilities:

- solve Schrödinger equation using a temperature-dependent heavy quark potential $V(r, T)$
- calculate quarkonium spectrum directly in finite T lattice QCD

2.2. Potential Models for Quarkonium Dissociation

- model heavy quark potential (Schwinger model)

$$V(r, T) = \sigma r \left\{ \frac{1 - e^{-\mu r}}{\mu r} \right\} - \frac{\alpha}{r} e^{-\mu r}$$

with screening mass $\mu(T) = 1/r_D(T)$

solve Schrödinger equation: with increasing T , bound state i disappears at some $\mu_i(T) = \mu(T_i)$

use screening mass from lattice estimates $\mu(T) \simeq 4 T$ for $T > 0$ to determine dissociation temperature T_i

charmonia:

$$\psi' \text{ and } \chi_c \text{ dissociated at } T \simeq T_c$$
$$J/\psi \text{ at } T \simeq 1.2 T_c$$

- determine potential $V(r, T)$ from lattice studies of heavy quark free energy: [Karsch, Kaczmarek, HS 2008]

Consider free energy with and without color singlet $Q\bar{Q}$ pair

Hamiltonian \mathcal{H}_0 for QGP with $Q\bar{Q}$:

$$F_Q(r, T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_Q/T\}$$

Hamiltonian \mathcal{H}_Q for QGP without $Q\bar{Q}$:

$$F_0(T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_0/T\}$$

lattice QCD: free energy difference $F(r, T) = F_Q(r, T) - F_0(T)$

internal energy difference $U(r, T)$ & entropy difference $S(r, T)$

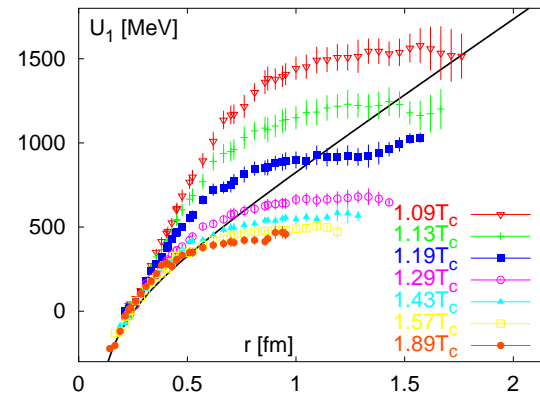
$$U(r, T) = -T^2 \left(\frac{\partial [F(r, T)/T]}{\partial T} \right) = F(r, T) + TS(r, T)$$

Internal energy difference $U(r, T) = \langle \mathcal{H}_Q(r, T) \rangle - \langle \mathcal{H}_0(T) \rangle$

for static heavy quarks, H_Q contains no kinetic term, so $U(r, T)$ gives change in potential energy due to presence of $Q\bar{Q}$ pair

at $T = 0$:
$$U(r, T = 0) = F(r, T = 0) = \sigma r - \frac{\alpha}{r}$$

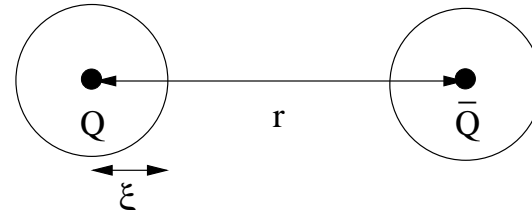
for $T > T_c$ & two flavor QCD, very much stronger interaction potential in the region $0.3 \leq r \leq 1.5$ fm



consider $Q\bar{Q}$ in QGP (above T_c)

– large distance limit

for $r \rightarrow \infty$, \exists polarization clouds,
of radius correlation length $\xi(T)$



– short distance limit

for $r \rightarrow 0$ (so that $r \ll T^{-1}$):

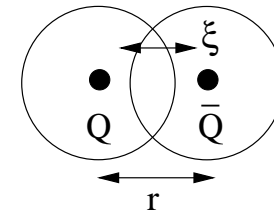
– $Q\bar{Q}$ neutralizes itself & does not see medium

– medium does not see color-neutral $Q\bar{Q}$;

– hence effectively $T = 0$ and $U(r, T) = -\frac{4\alpha(r)}{3r}$

– intermediate separation regime

at small r , polarization clouds overlap



how does this affect binding?

$U(r, T)$ is sum of $Q\bar{Q}$ interaction and “cloud” energy

concentrate on binding (remove constant $U(r \rightarrow \infty, T)$),

consider effective coupling $\alpha(r, T) = \frac{3}{4} r^2 \left(\frac{\partial U(r, T)}{\partial r} \right)$

at $T = 0$: $\alpha(r, T = 0) = -\alpha + \sigma r^2$

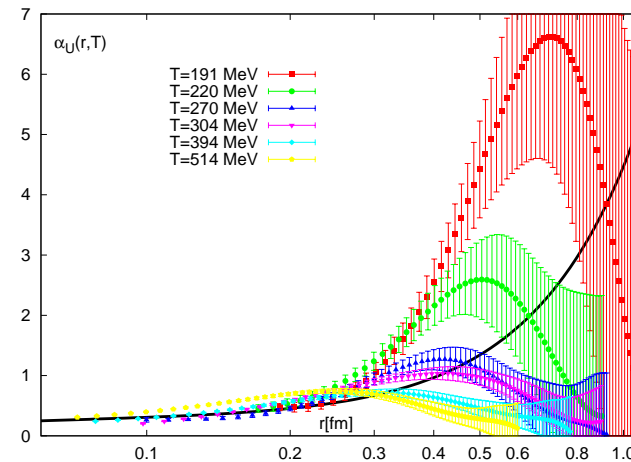
result for QGP:

strong enhancement

when $T_c < T < 2 T_c$

effective binding in medium

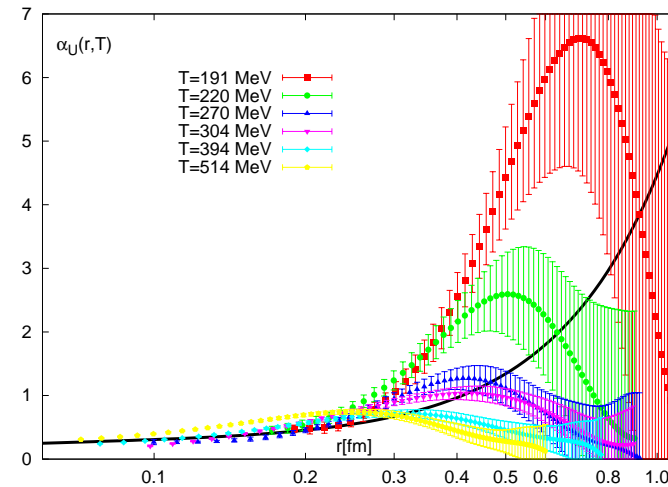
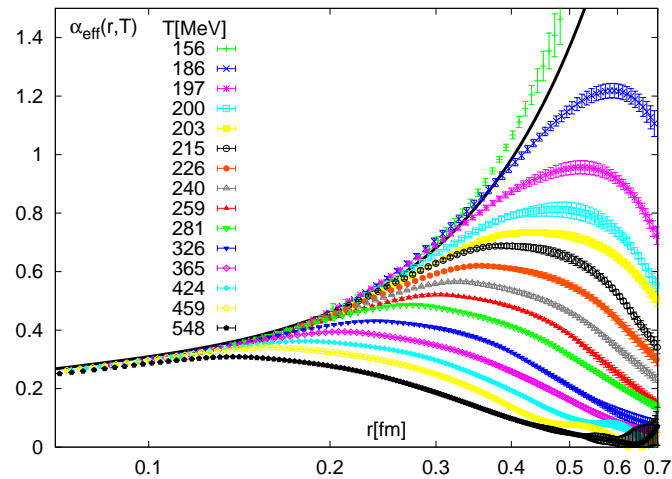
is stronger than in vacuum



\exists additional binding mechanism in medium

when polarization clouds overlap
 \exists “cloud-cloud” binding
 in addition to direct $Q\bar{Q}$ binding

to include cloud-cloud binding, must use $U(r, T) = V(r, T)$ in Schrödinger equation; compare to $F(r, T)$:



illustrate: dissociation in semi-classical approximation

$$2m_c + \frac{p^2}{m_c} + V(r, T) = M$$

uncertainty relation $\Rightarrow p^2 \simeq c/r^2$,

$$2m_c + \frac{c}{m_c r^2} + V(r, T) = M$$

minimize

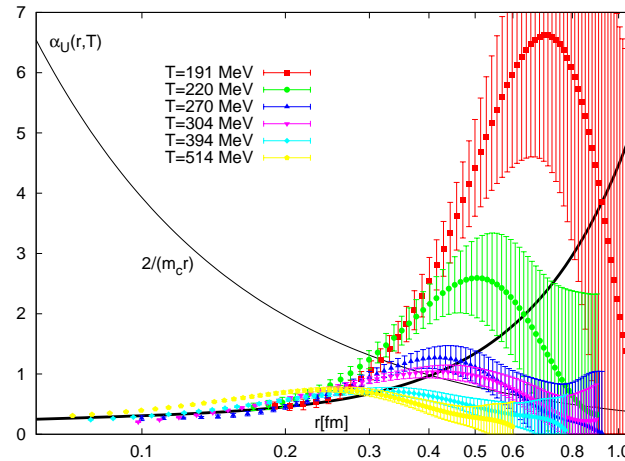
$$E(r, T) = \frac{c}{m_c r^2} + V(r, T)$$

to get

$$\frac{2c}{m_c r} = r^2 \left(\frac{\partial V(r, T)}{\partial T} \right) = \frac{4}{3} \alpha(r, T)$$

fix c by $M = M_{J/\psi}$,

compare kinetic term $2c/m_c r$ to potential term $\alpha_U(r, T)$



$\Rightarrow J/\psi$ dissociation at about $1.5 - 2 T_c$

more general: solve
Schrödinger equation

ψ' dissociated at $T \simeq 1.1 T_c$

χ_c dissociated at $T \simeq 1.2 T_c$

J/ψ survives up to $T \gtrsim 2 T_c$

- reason for later dissociation: $U(r, T)$ provides stronger binding than Schwinger model potential

ambiguity in specifying “lattice” potential: $U(r, T)$ or $F(r, T)$

$F(r, T)$ results \sim Schwinger model

various alternatives: $V(r, T) = a F(r, T) + b U(r, T)$ reduce binding, lower dissociation temperatures

resolution: determine dissociation points directly in finite T lattice QCD

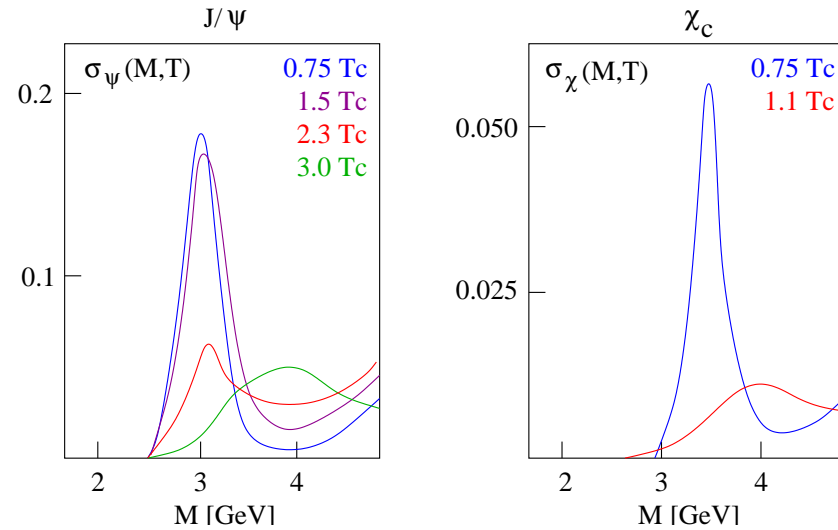
2.3 Lattice Studies of In-Medium Charmonium Survival

determine $c\bar{c}$ spectrum $\sigma(\omega, T)$ in specific quantum number channel
as function of $c\bar{c}$ energy ω

results for quenched and unquenched ($N_f = 2$) QCD agree

schematic pattern

lattice resolution limits
precision; reliable only
for resonance peak strength,
position, not width,
not continuum ($\omega > 4$ GeV)



charmonia

χ_c is dissociated for $T \geq 1.1 T_c$
 J/ψ persists up to $1.5 T_c < T < 2.3 T_c$

preliminary conclusion: – higher excited states melt near T_c
– J/ψ melts around $2 T_c$

in accord with direct lattice calculation and potential model studies

- caveat: physical widths

3. Dynamics of Quarkonium Dissociation

Study of global medium effects on quarkonia

⇒ only hot deconfined medium can dissociate ground state

deconfined medium: constituents are unbound partons

confined medium: constituents are hadronic “comovers”

why cannot collisions with hadrons dissociate J/ψ ?

Collision Dissociation of Quarkonia

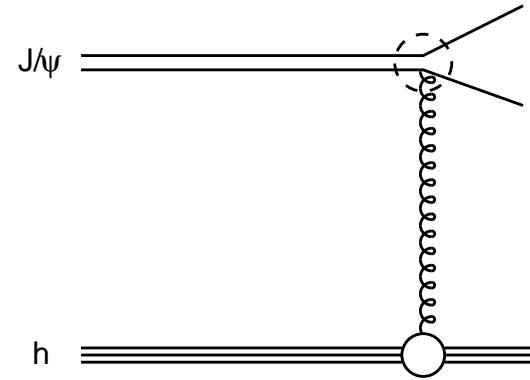
consider J/ψ dissociation:

- J/ψ is small ($r_{J/\psi} \sim 0.2$ fm), resolved only by hard probes
- J/ψ is tightly bound ($2M_D - M_{J/\psi} \sim 0.6$ GeV), dissociated only by hard probes

how could hadrons interact with J/ψ ?

J/ψ interacts with gluon in hadron

gluon momentum distribution (PDF)
in hadron, $g(x)$, with $x = 2k/\sqrt{s}$,
as determined in DIS



for pions, $g(x) \sim (1 - x)^3$, hence

$$\langle k_g \rangle_h \simeq \frac{1}{5} \langle p_h \rangle$$

in confined matter, $\langle p_h \rangle \sim 3T$, with $T < 175$ MeV:

$$\langle k_g \rangle_h \simeq \frac{3}{5} T \leq 0.1 \text{ GeV} \ll \text{binding energy } 0.6 \text{ GeV}$$

\Rightarrow dissociation in hadronic matter impossible: gluons in hadronic
constituents of confined matter are too soft to dissociate J/ψ

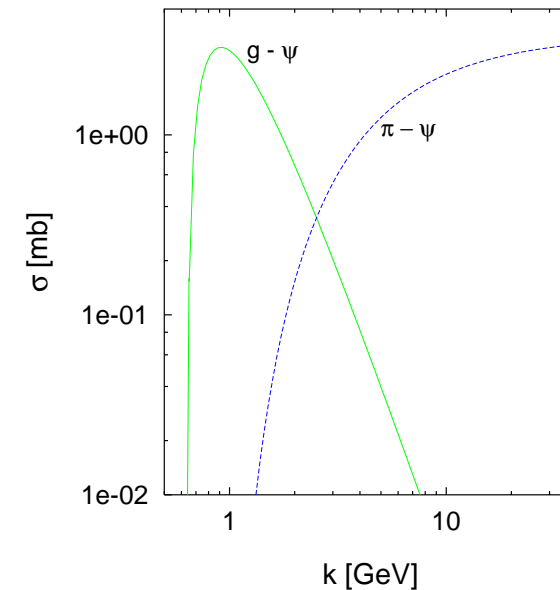
in deconfined medium,

$$\langle k_g \rangle \simeq 3 T$$

so that for $T \geq 1.2 T_c$,
enough energy to
overcome J/ψ binding

More quantitative:

gluon dissociation (QCD photo effect)



4. Quarkonium Production in Nuclear Collisions

Aim: probe medium produced in nuclear collisions by studying the
fate of quarkonia

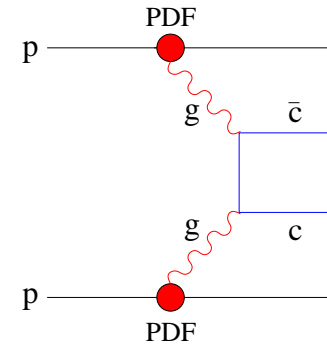
How to produce the charmonium to be put into the medium?

4.1 Quarkonium Production in Hadronic Collisions

$c\bar{c}$ production:

hard process calculated in perturbative QCD

with PDF determined from DIS



fixed fraction of subthreshold $c\bar{c}$ production \Rightarrow charmonium

colour evaporation model

$$\sigma_{hh \rightarrow J/\psi}(s) = f_{J/\psi} \sigma_{hh \rightarrow c\bar{c}}(M_{c\bar{c}} \leq 2M_D; s)$$

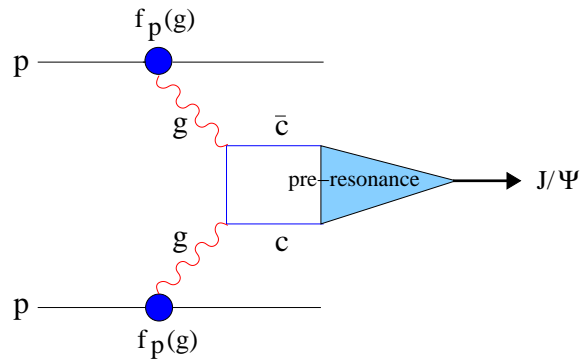
with $\sqrt{s} \sim$ cms collision energy

energy-independent fractions f_i for all charmonium states i

(similarly for $b\bar{b}$ and bottomonium)

\Rightarrow correct size & energy dependence of all quarkonium cross sections

J/ψ formation and time scales:



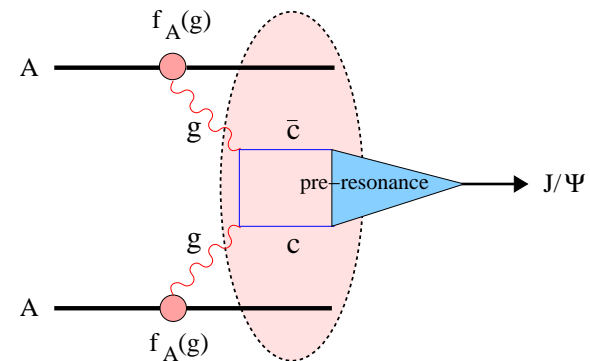
	0.05 fm	0.25 fm
hard	pre-resonance	resonance
	$\tau_{c\bar{c}} = 1/2m_c$	$\tau_g = 1/\sqrt{2}m_c \Lambda_{\text{qcd}}$

in hadronic collisions, hadron PDF's and $c\bar{c}$ formation in vacuum

4.2 Quarkonium Production in Nuclear Collisions

nuclear collisions:

- modified PDF's
 shadowing/antishadowing
- evolution in nuclear matter
 pre-resonance absorption



hence J/ψ production is modified in nuclear medium,
independent of/before any QGP formation and QGP effects
 \Rightarrow must take that into account before looking for QGP effects

How?

both in theory and in experiment:

dilepton, open charm and charmonium production
in pA/dA collisions

essential for any understanding of quarkonium production in nuclear collisions

Procedure:

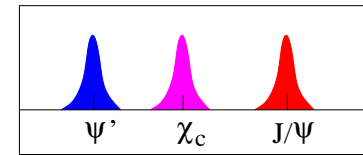
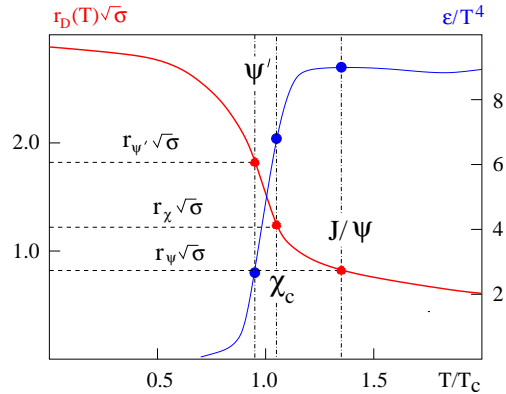
- determine PDF's in nuclear matter by open charm & dilepton production in pA/dA in the relevant kinematic regions;
- predict production of the different charmonium states via colour evaporation model;
- determine pre-resonance absorption by J/ψ & ψ' production in pA/dA in the relevant kinematic regions (Glauber analysis).

Assume: PDF modifications, nuclear absorption, pre-resonance effects are accounted for; what remains?

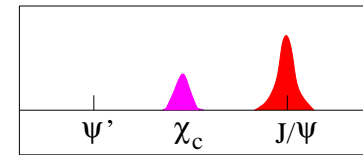
4.3 Sequential Quarkonium Suppression

J/ψ production in hadronic collisions:

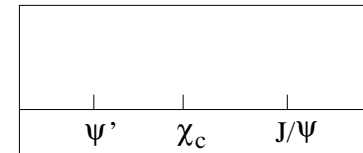
- $\sim 60\%$ direct $J/\psi(1S)$ production
- $\sim 30\%$ decay $\chi_c(1P) \rightarrow J/\psi + x$
- $\sim 10\%$ decay $\psi'(2S) \rightarrow J/\psi + x$



$T < T_c$



$T \sim 1.1 T_c$



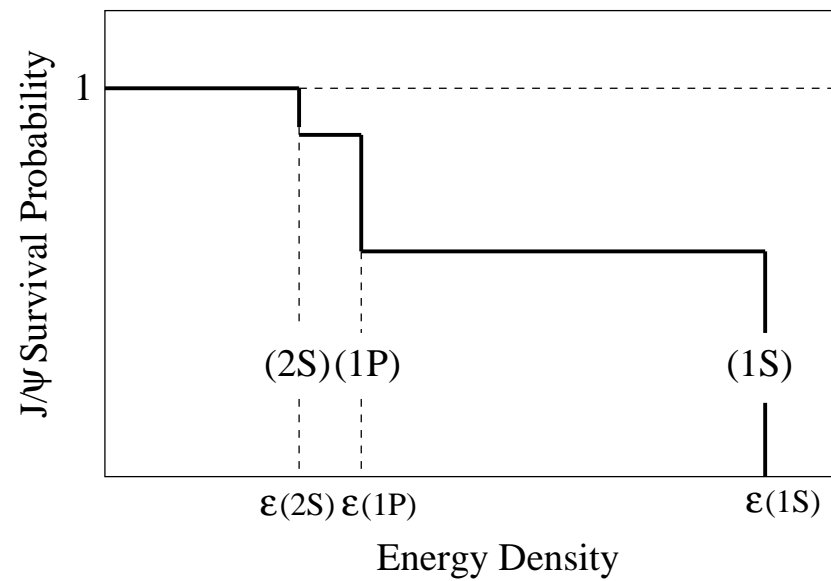
$T \gg T_c$

In a thermal QCD medium, higher excited states are absorbed at lower temperatures, energy densities: first ψ' , then χ_c , last J/ψ

Hence: if

- nuclear collisions produce a thermal QCD medium, and
- nuclear/pre-resonance effects on charmonium production can be accounted for

then J/ψ suppression should be observed in sequential form



with suppression onsets and amounts predicted by QCD

4.4 Charmonium Regeneration

so far: charmonia as “outside” probes, pre-QGP production
no thermal charmonium production in QGP:

$$\exp\{-M_{J/\psi}/T_c\} \sim 10^{-8}$$

but what if initial QGP contains more than thermal amount of charm quarks?

- $c\bar{c}$ production is **hard** process $\sim N_{coll}$, in contrast to **soft** hadron (u, d, s) production $\sim N_{part}$
[breaks down at high energy, parton saturation]
- increase collision energy \rightarrow **increase charm content** in produced system
- c or \bar{c} from a given nucleon-nucleon collision can at hadronization bind with charm constituents from different collisions
 \exists new **exogamous** charmonium production mechanism;
 c and \bar{c} in such charmonia have **different parents**,
in contrast to **introgamous** production in pp

Predict:

High energy \Rightarrow **enhanced J/ψ production** in AA re pp

Scenario:

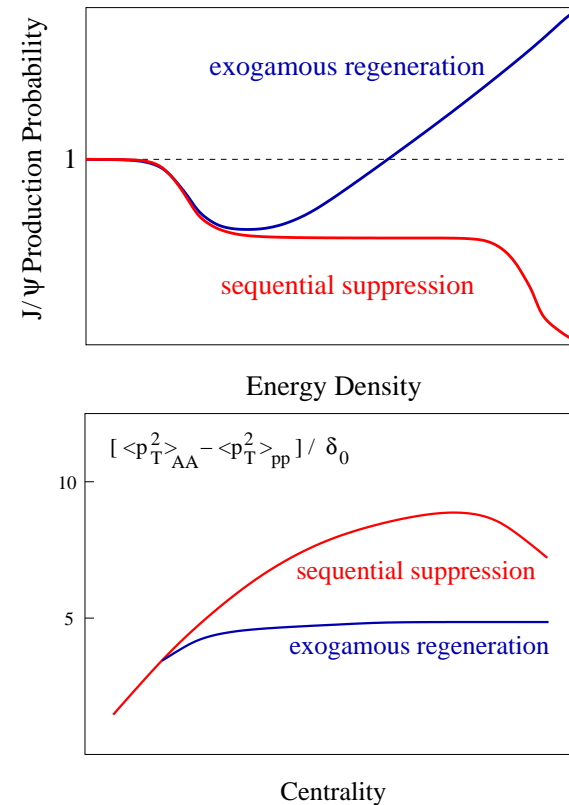
- at sufficiently high energy, charmonia from primary production (intragamous) melt in QGP;
- QGP is over-saturated in charm (higher charm abundance than in thermal equilibrium), due to charm production \sim number of collisions;
- at hadronization, statistical combination (regeneration) of $c\bar{c}$ pairs from the set of all (exogamous) charm quarks in QGP more than compensates the removal of primary production;
- at high energy, J/ψ enhancement, not suppression

How to distinguish between

- J/ψ suppression in equilibrium QGP
- J/ψ enhancement by charm over-abundance?

Remove nuclear matter & (at high energy) beauty decay effects

- overall J/ψ survival:
suppression vs. enhancement
at high energy densities
- p_T behaviour:
initial state parton scattering
vs. final state charm production



- in general, regeneration \rightarrow quarkonium momentum distributions
as convolution of open charm momenta

5. Conclusions

In statistical QCD, quarkonium spectra provide an unambiguous tool to determine temperature/energy density of QGP.

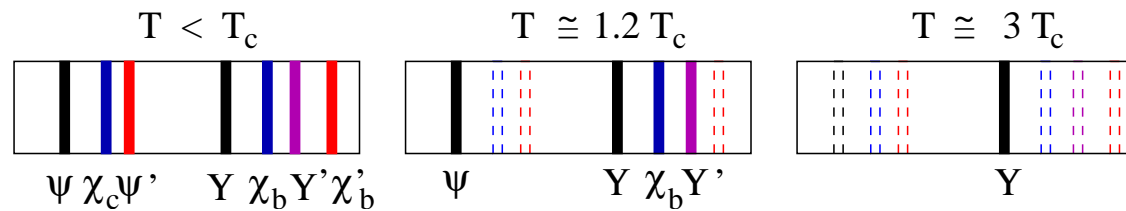
In nuclear collisions?

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In nuclear collisions?

- If J/ψ production remains “external” probe (no regeneration), sequential suppression leads to a quantitative QCD prediction

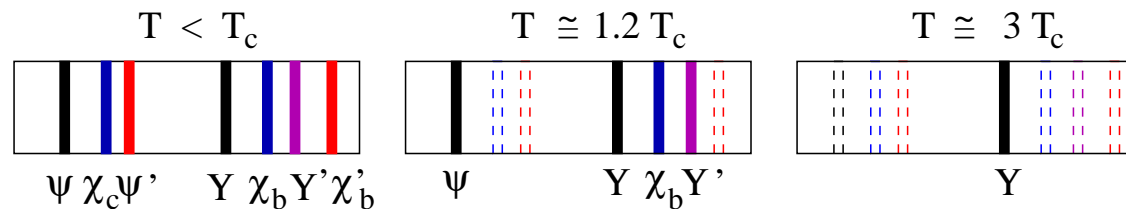


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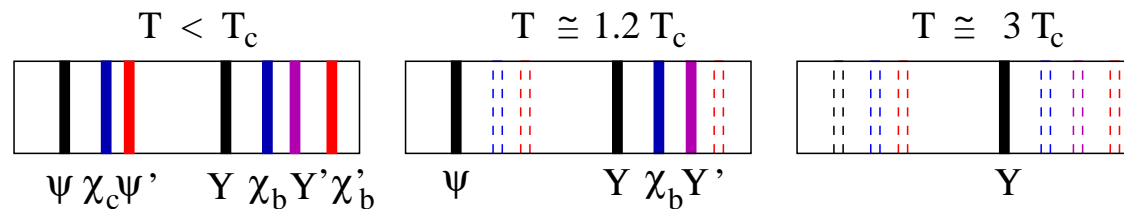
- If there is regeneration & J/ψ enhancement: clear evidence of thermalization on a pre-hadronic level, but no thermometer.

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In nuclear collisions?

- If J/ψ production remains “external” probe (no regeneration), sequential suppression leads to a quantitative QCD prediction



- If there is regeneration & J/ψ enhancement: clear evidence of thermalization on a pre-hadronic level, but no thermometer.
- In that case, bottomonium production remains as the QGP thermometer.

There is much interesting work left to do...