The Analysis of Matter in QCD

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Lecture 0: The Thermodynamics of Quarks and Gluons

Lecture 1: The States of Matter in QCD

Lecture 2: The Spectral Analysis of the QGP

Lecture 3: The Origin of Thermal Hadron Production

The Origin of
Thermal Hadron Production

basic observation in all high energy multihadron production

thermal production pattern

Fermi, Landau, Pomeranchuk, Hagedorn

- species abundances $\sim$ ideal resonance gas at T_H
- universal $T_H \simeq 150 - 200$ MeV for all (large) $\sqrt{s}$
- thermal transverse momentum spectra with same T_H

caveats: baryon density, strangeness, heavy flavors, flow

begin by recalling what is "thermal" and what
are the experimental features to be described

1. Thermal Hadron Production

what is "thermal"?

- equal *a priori* probabilities for all states in accord with a given local average energy ⇒ temperature T;
- grand canonical partition function of ideal resonance gas

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3}\, \phi(m_i, T)$$

- Boltzmann factor

$$\phi(m_i, T) = \int d^3p\, \exp\{\sqrt{p^2 + m_i^2}/T\} \sim \exp -(m_i/T);$$

- uniform rapidity distribution of identical fireballs
- relative abundances $\dfrac{N_i}{N_j} = \dfrac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)}$

- transverse momenta $$\frac{dN}{dp_T^2} \sim \exp -\frac{1}{T}\sqrt{m_i^2 + p_T^2}.$$

Hadronization features in elementary & nuclear collisions

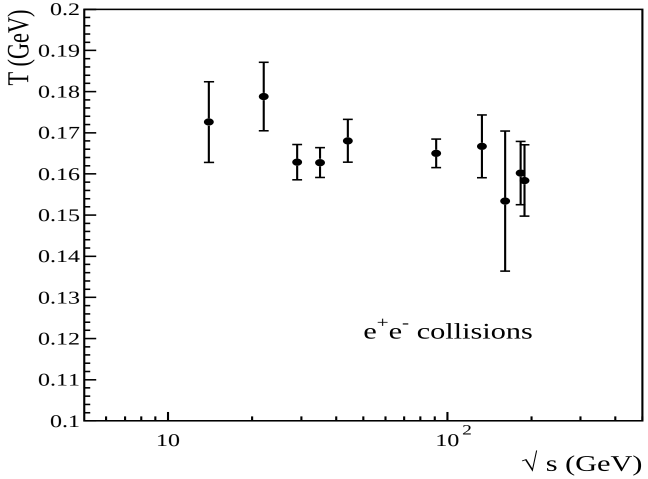
- jet structure
- multiplicity per jet $\ln \sqrt{s}$
  (caveat multi-jets, evolution)
- universal hadronization temperature
  – from species abundances
  (caveat: strangeness, heavy flavors)
  – from transverse mass spectra
  (caveat: flow in nuclear collisions)
- initial state quantum number structure
  – baryon number, heavy flavor

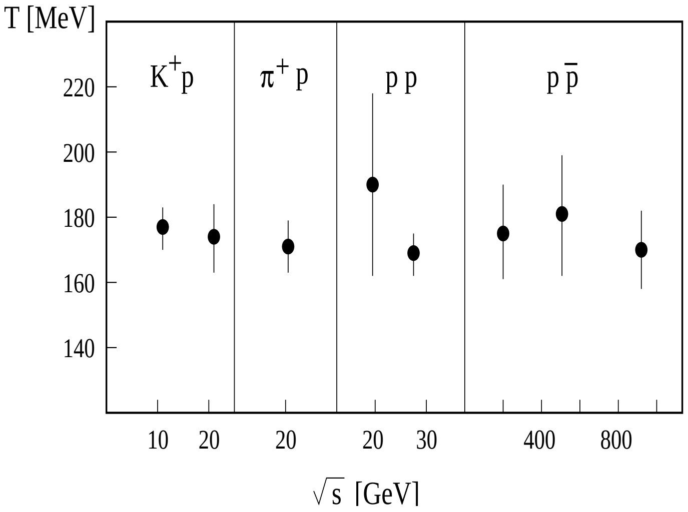
$\exists$ a universal scenario (including the caveats)?

summarize experimental situation on abundances & p_T

Species abundances in elementary collisions

[Becattini et al. 1996 - 2008]





Conclude:

$$\boxed{T_H = 170 \pm (10-20)\ \mathrm{MeV};\ \gamma_s \simeq 0.5 - 0.7}$$

independent of $\sqrt{s}$, incident production configuration

Transverse momentum spectra in elementary collisions

requires resonance decay code; model dependence?

[Becattini & Passaleva 2001]

pp at $\sqrt{s} = 27.4$ GeV:

average $T = 163$ MeV

similar analyses for

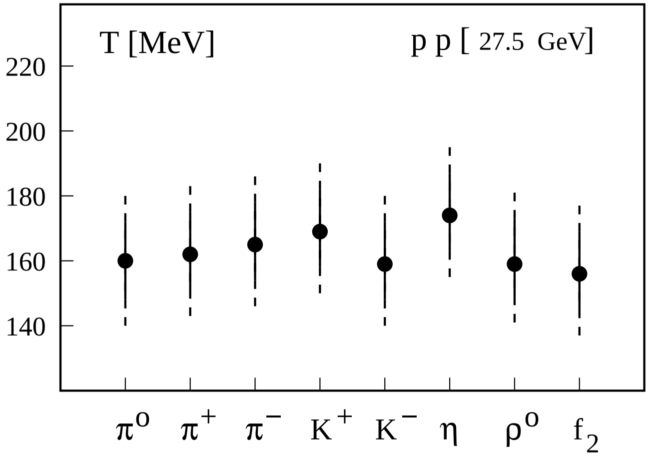
K^+p at $\sqrt{s} = 21.7$ GeV:

average $T = 165$ MeV

π^+p at $\sqrt{s} = 21.7$ GeV:

average $T = 160$ MeV

from transverse momentum spectra



Conclude:

$$\boxed{T_H = 163 \pm ? \text{ MeV}}$$

independent of species

Heavy ion collisions

- temperature T, baryochem. pot. μ_B; $\mu_B \Downarrow$ for $\sqrt{s} \Uparrow$
- elementary high energy collisions low baryon content
- compare to species abundances for RHIC, peak SPS

SPS (Pb-Pb), $\sqrt{s} = 17$ GeV

$$T_H = 157.8 \pm 2.5 \text{ MeV}, \ \mu_B = 248.9 \pm 9.0 \text{ MeV}$$

RHIC (Au-Au), $\sqrt{s} = 130$, $y = 0$ GeV

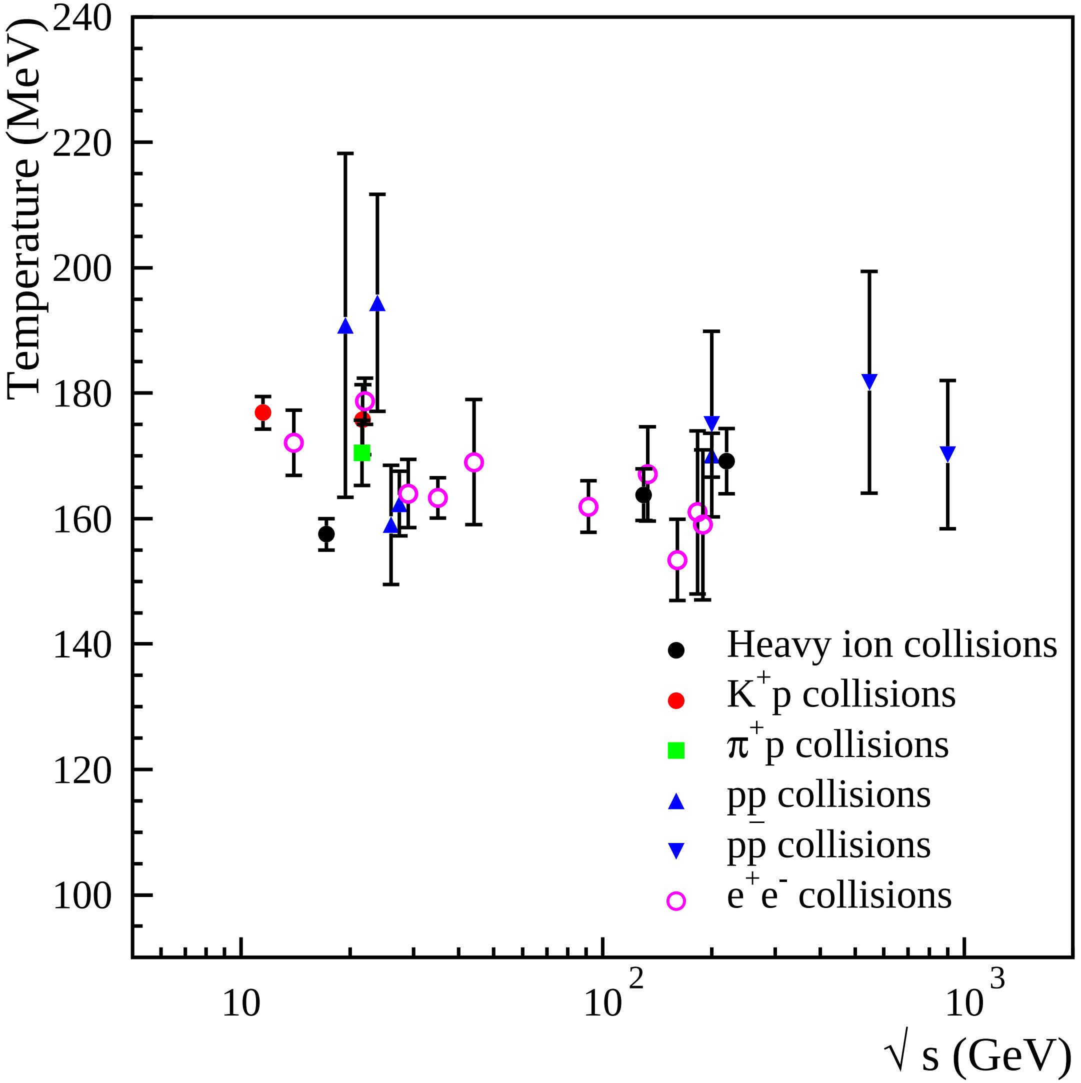
$$T_H = 163.8 \pm 4.1 \text{ MeV}, \ \mu_B = 36.3 \pm 10.2 \text{ MeV}$$

RHIC (Au-Au), $\sqrt{s} = 200$ GeV

$$T_H = 169.2 \pm 5.2 \text{ MeV}, \ \mu_B = 29.5 \pm 11.2 \text{ MeV}$$

in general $\gamma_s \simeq 0.8 - 1.1$

[Andronic, Braun-Munzinger & Stachel 2006, Becattini & Manninen 2008]


Temperature (MeV)
240
220
200
180
160
140
120
100
10
10^2
10^3
$\sqrt{s}$ (GeV)
● Heavy ion collisions
● K^+p collisions
■ π^+p collisions
▲ pp collisions
▼ $p\bar{p}$ collisions
○ e^+e^- collisions

Conclude:

The hadron abundances in all high energy collisions (e^+e^- annihilation, hadron-hadron & nuclear collisions) are specified by an ideal resonance gas of a universal temperature

$$T_H \simeq 170 \pm 20 \text{ MeV.}$$

The transverse momentum spectra in elementary collisions are in accord with such "thermal" behavior; ∃ broadening (flow) in nuclear collisions.

Strangeness production in elementary collisions is systematically reduced; strangeness suppression is weakened or removed in nuclear collisions.

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WHY?

Why should high energy collisions show thermal behavior?

How is a thermal state attained ?

Conventional Approach

- kinetic theory, Boltzmann equation
- many particles, finite collision cross section, sufficient evolution time
- arbitrary starting configuration of particles, collisions and evolution towards maximum entropy, equilibration time to attain stable Boltzmann distribution.

this approach has determined most thinking about thermal behavior in QCD up to today:

parton collisions & equilibration, hadronization

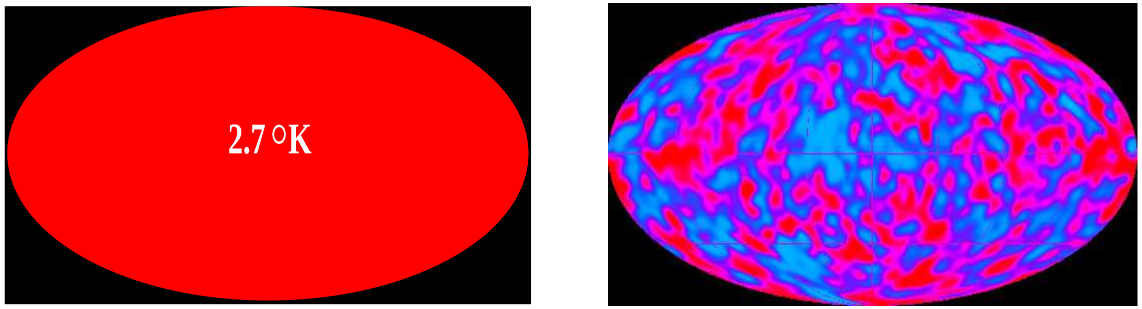
Multiple parton interactions → kinetic thermalization?

Is this really possible in high energy collisions evolving in time?

or $\exists$ a "non-kinetic" mechanism producing statistical features?

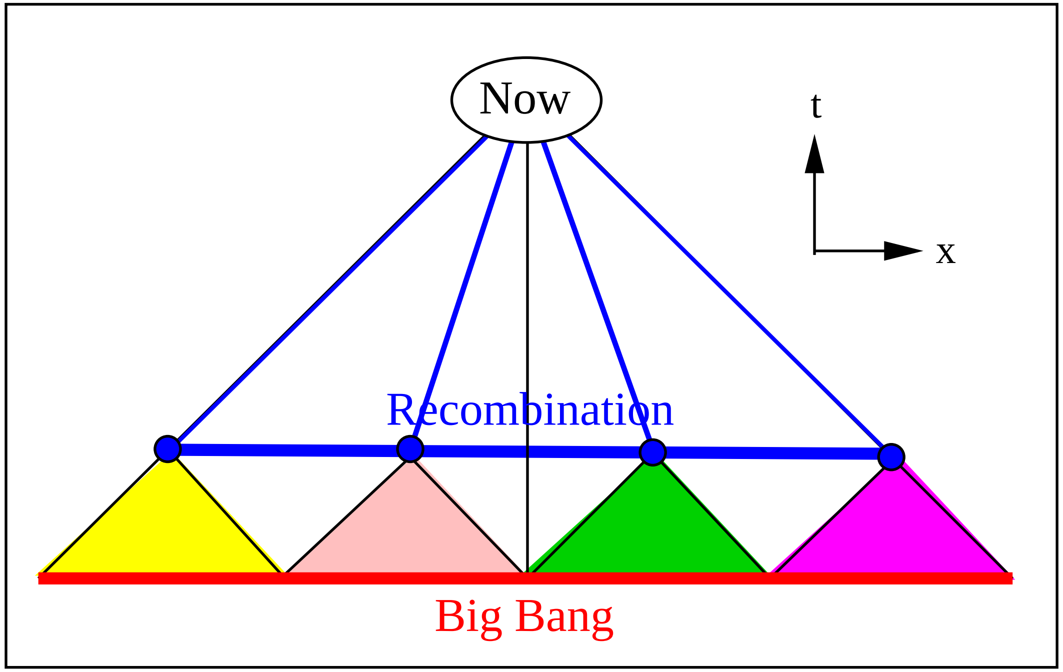
Prelude: Cosmic Thermalization

microwave background radiation



visible universe seen with low (10^{-2}) and with high (10^{-5}) resolution

radiation from the end of the "recombination era":
photons at $T \simeq 3000°\mathrm{K}$, cosmic redshift $\longrightarrow$ 2.7° K



- same CBR temperature measured from regions of the Universe causally disconnected when CBR formed
- So how was equilibrium created?
- why does the orchestra play the same melody in tune if the players cannot communicate?

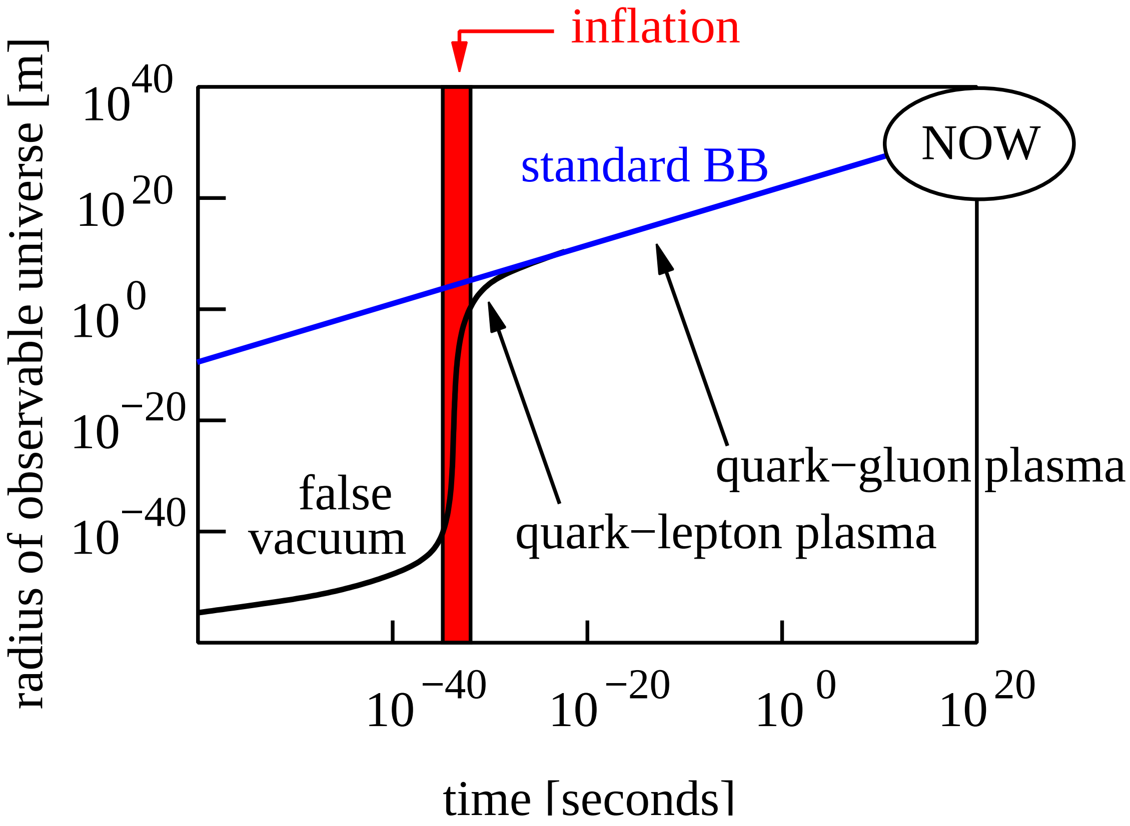
Alan Guth: The Purple Creatures

One can pretend, for the sake of discussion, that the Universe is populated by little purple creatures, each equipped with a furnace and a refrigerator, and each dedicated to the cause of creating a uniform temperature.

Those little creatures, however, would have to communicate at roughly 100 times the speed of light if they are to achieve their goal of creating a uniform temperature across the visible Universe by 300,000 years after the Big Bang. Since nothing can transmit energy faster than light, that cannot account for the uniformity.

The classical form of Big Bang theory requires us to postulate, without explanation, that the primordial fireball filled space from the beginning. The temperature was the same everywhere by assumption, not as a consequence of any physical process.

The solution proposed by Guth: inflation

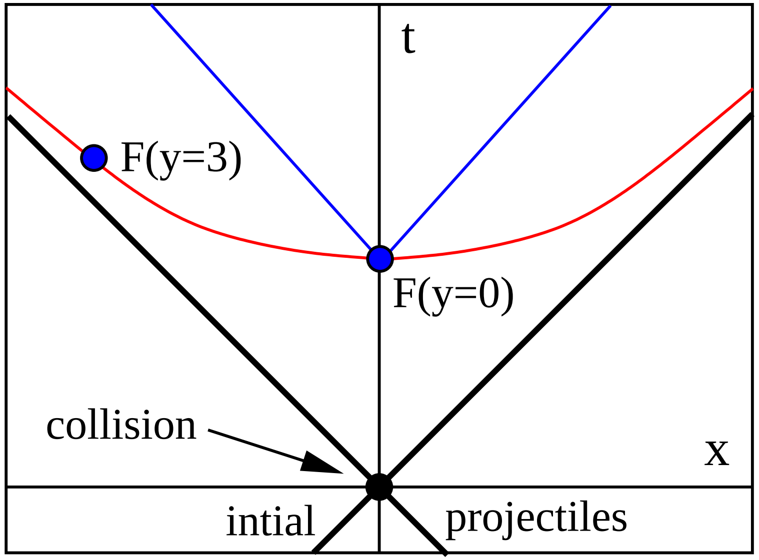


pre-inflationary “medium” is hot & equilibrated;

quarks & leptons in early universe “born in equilibrium”.

equilibration of fireballs produced in high energy collisions

equilibration at fixed $\tau = \sqrt{t^2 - x^2} = \tau_0$



equilibration points of fireballs at rapidities **y=0** and at **y=3**

are causally disconnected: again ∃ horizon problem

thermal behavior must somehow arise **locally**

$\exists$ a "non-kinetic, local" mechanism producing statistical features?

$\exists$ a common origin of statistical hadron production in all high energy collisions?

$\mathcal{R}$ussian $\mathcal{F}$olklore:

Passing color charge disturbs vacuum,
vacuum recovers locally,
by producing hadrons according to maximum entropy.

What does that mean?

Confinement $\Rightarrow$ *Event Horizon* $\Rightarrow$ *Unruh Radiation*

[Castorina, Kharzeev, HS 2007]

2. Event Horizons & Hawking-Unruh Radiation

- Unruh radiation [Unruh 1976]

event horizon arises for systems in uniform acceleration

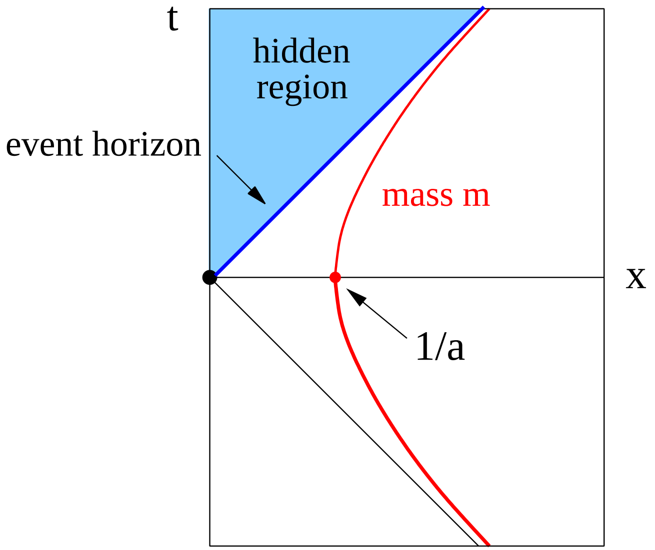
mass m in uniform acceleration a

$$\frac{d}{dt}\frac{mv}{\sqrt{1-v^2}} = F$$

$$v = dx/dt,\ F = ma,\ c = 1$$

solution: hyperbolic motion

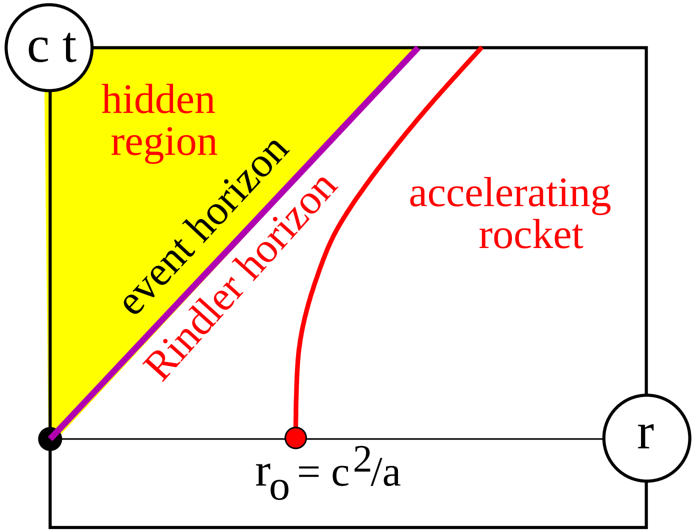
$$x = \frac{1}{a}\cosh a\tau \quad t = \frac{1}{a}\sinh a\tau$$

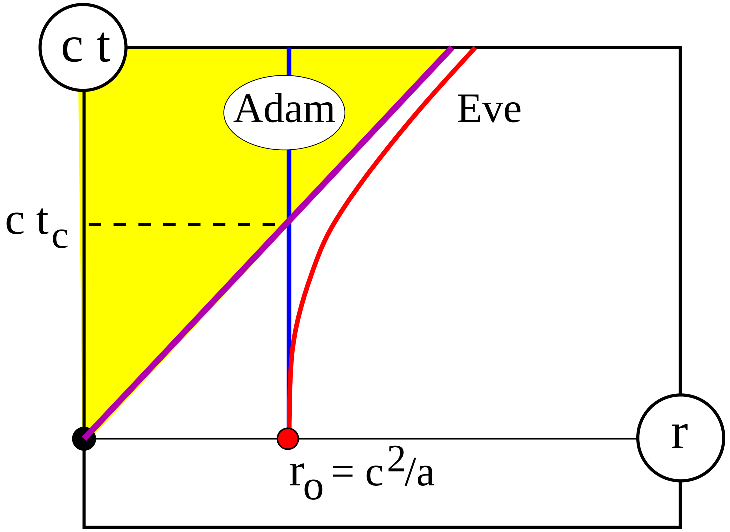


∃ event horizon: m cannot reach hidden region
observer in hidden region cannot communicate with m

event horizon: defines causal future for observer at $r=0$

Rindler horizon: defines communication limit for rocket





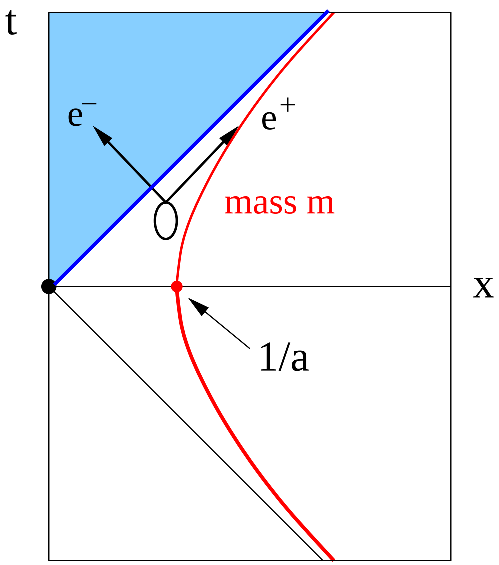
Adam and Eve: Adam remains, Eve leaves with rocket

after t_c, Adam can no longer send message to Eve

Eve can send message to Adam, but will never get answer: for her, he's in a black hole (beyond her Rindler horizon)

Entanglement of Adam and Eve is destroyed

m passes through vacuum, can use part of acceleration energy to excite vacuum fluctuations on-shell



e^+ absorbed in detector on m
e^- disappears beyond event horizon

equivalent:
e^- tunnels through event horizon

broken “quantum entanglement”
$\sim$ Einstein-Podolsky-Rosen effect

observer on m as well as observer in hidden region have incomplete information: $\Rightarrow$ each sees thermal radiation

observer on m:
physical vacuum $\sim$ thermal medium of temperature T_U

observer in hidden region:
passage of $m \longrightarrow$ thermal radiation of temperature T_U

Unruh temperature

$$T_U = \frac{\hbar a}{2\pi c}$$

relativistic (c) quantum ($\hbar$)effect

Unruh temperature $$T_U = \frac{\hbar a}{2\pi c}$$

relativistic (c) quantum ($\hbar$)effect

Applications

- Black Holes

event horizon $R = 2GM$ (Schwarzschild radius)

$$F = ma = G\frac{Mm}{R^2} \;\Rightarrow\; a = \frac{GM}{R^2} = \frac{1}{4GM}$$

$$\Rightarrow\; T_U = \frac{a}{2\pi} = \frac{1}{8\pi GM} = T_{BH}$$

obtain temperature T_{BH} of Hawking radiation

[Hawking 1975]

- Schwinger Mechanism

  in strong electric field $\mathcal{E}$, vacuum becomes unstable against pair production

  $F = e\mathcal{E} = (m/2)a$ leads to production of pair of charges of mass m

$$T_U = \frac{a}{2\pi} = \frac{e\mathcal{E}}{\pi m}$$

$$P(m,\mathcal{E}) \sim \exp\{-m/T_U\} = \exp\{-\pi m^2/e\mathcal{E}\}$$

  obtain Schwinger production probability $P(m,\mathcal{E})$

  [Schwinger 1951]

In general: [T. D. Lee 1986, Parikh & Wilczek 2000]

event horizon ∼ information transfer forbidden

⇒ quantum tunnelling ∼ thermal radiation

3. Pair Production and String Breaking

Basic process:

two-jet e^+e^- annihilation, cms energy $\sqrt{s}$:

$$e^+e^- \longrightarrow \gamma* \longrightarrow q\bar{q} \longrightarrow \text{ hadrons}$$

$q\bar{q}$ separate subject to constant confining force $F = \sigma$

initial quark velocity $v_0 = \dfrac{p}{\sqrt{p^2 + m^2}}$, $p \simeq \sqrt{s}/2$

Solve $ma = \sigma$ (hyperbolic motion): [Hosoya 1979, Horibe 1979]

$$\tilde{x} = [1 - \sqrt{1 - v_0\tilde{t} + \tilde{t}^2}] \ , \ \tilde{x} = x/x_0 \ , \ \tilde{t} = t/x_0$$

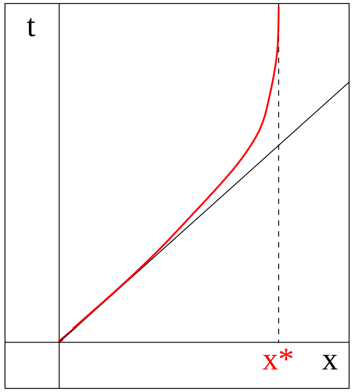
with $$x_0 = \frac{m}{\sigma}\frac{1}{\sqrt{1 - v_0^2}} = \frac{m}{\sigma}\,\gamma = \frac{1}{a}\,\gamma$$

classical turning point $v(t^*) = 0$ at

$$x^* = x(t^*) = \frac{m}{\sigma}\gamma\,[1-\sqrt{1-(v_0/2)^2}] \simeq \frac{\sqrt{s}}{2\sigma}$$

$q\bar{q}$ can separate arbitrarily far
if $\sqrt{s}$ is large enough

What's wrong?



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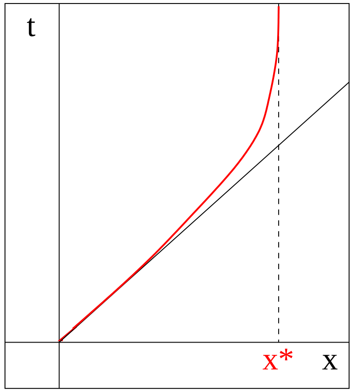
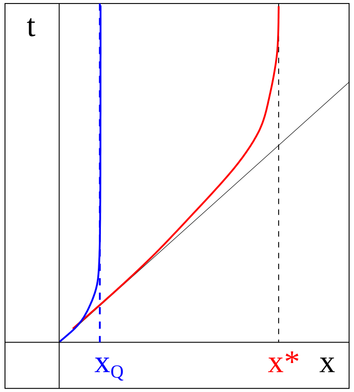


What's wrong?

classical event horizon

Strong field ⇒ vacuum unstable
against pair production [Schwinger 1951]

when $\sigma x > \sigma x_Q \equiv 2m$
string connecting $q\bar{q}$ breaks

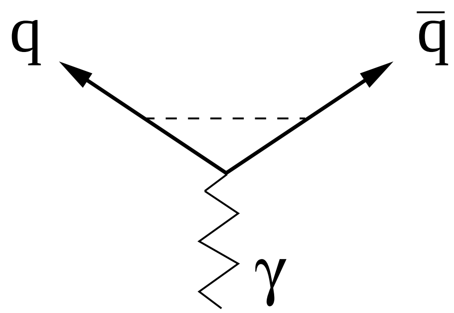


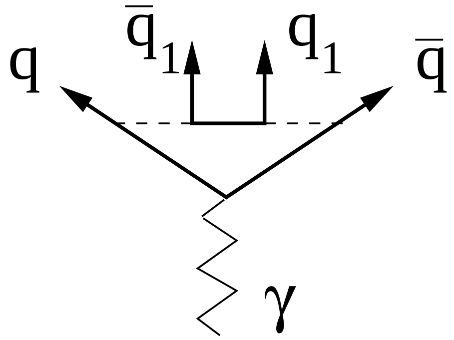
Result:

quantum event horizon

Hadron production in e^+e^- annihilation:

“inside-outside cascade” [Bjorken 1976]





$q\bar{q}$ flux tube has thickness
$$r_T \simeq \sqrt{\frac{2}{\pi\sigma}}$$

$q_1\bar{q}_1$ at rest in cms, but
$$k_T \simeq \frac{1}{r_T} \simeq \sqrt{\frac{\pi\sigma}{2}}$$

$q\bar{q}$ separation at $q_1\bar{q}_1$ production
$$\sigma x(q\bar{q}) = 2\sqrt{m^2 + k_T^2}$$

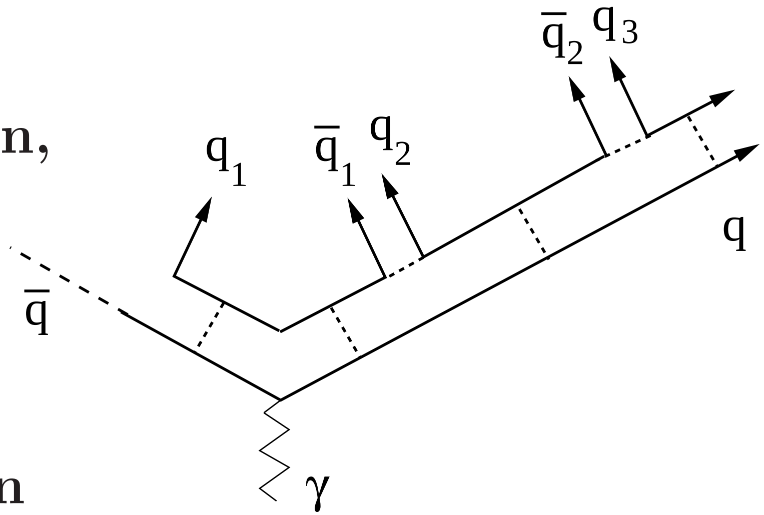
q_1 screens $\bar{q}$ from q, hence string breaking at

$$x_q \simeq \frac{2}{\sigma}\sqrt{m^2 + (\pi\sigma/2)} \simeq \sqrt{2\pi/\sigma} \simeq 1 \text{ fm}$$

new flux tubes $q\bar{q}_1$ and $\bar{q}q_1$
stretch $q_1\bar{q}_1$
to form new pair $q_2\bar{q}_2$

$$\sigma x(q_1\bar{q}_1) = 2\sqrt{m^2 + k_T^2}$$

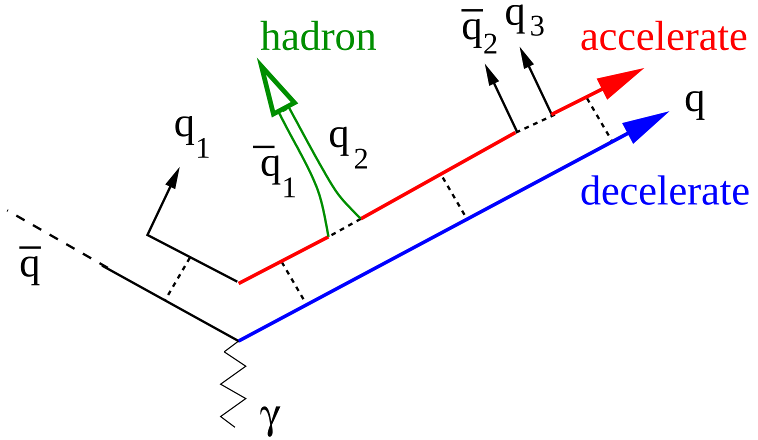
equivalent:
$\bar{q}_1$ reaches $q_1\bar{q}_1$ event horizon,
tunnels to become $\bar{q}_2$

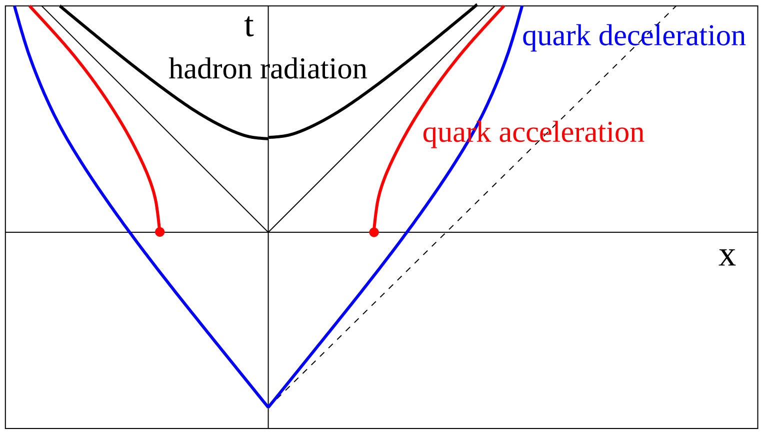


emission of hadron $\bar{q}_1 q_2$
as Hawking-Unruh radiation

self-similar pattern:

- **screening**
- **string breaking**
- **tunnelling**
- **quark acceleration /deceleration**
- **Hawking-Unruh radiation**





temperature of H-U radiation: what acceleration?

$(\bar{q}_1 \longrightarrow \bar{q}_2 \longrightarrow \bar{q}_3 \longrightarrow \ldots)$

$$a = F/m \ \Rightarrow \ a_q = \frac{\sigma}{w_q} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}}$$

string breaking & thickness determine $k_q \simeq \sqrt{\pi\sigma/2}$

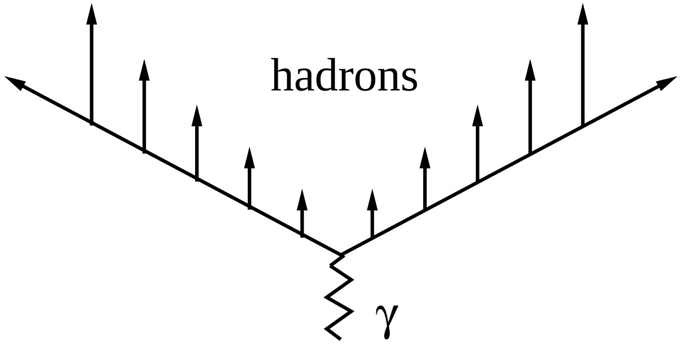
$$\Rightarrow \quad a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}}$$

for light quarks, $m_q \ll \sqrt{\sigma} \simeq 420$ MeV, hence

$$\boxed{T = \frac{a}{2\pi} \simeq \sqrt{\frac{\sigma}{2\pi}} \simeq 170 \text{ MeV}}$$

temperature of hadronic Hawking-Unruh radiation hadronization pattern:

hadron multiplicity?



thickness of classical “overstretched” string:

$$R_T^2 = \frac{2}{\pi\sigma}\sum_{k=0}^{K}\frac{1}{2k+1} \simeq \frac{2}{\pi\sigma}\ln 2K \simeq \frac{2}{\pi\sigma}\ln\sqrt{s}$$

quantum breaking at $x_q \sim r_T$, hence hadron multiplicity

$$\boxed{\nu(s) \simeq \frac{R_T^2}{r_T^2} \simeq \ln\sqrt{s}}$$

NB: parton evolution (minijets), multiple jets lead to stronger increase

4. Strangeness Production

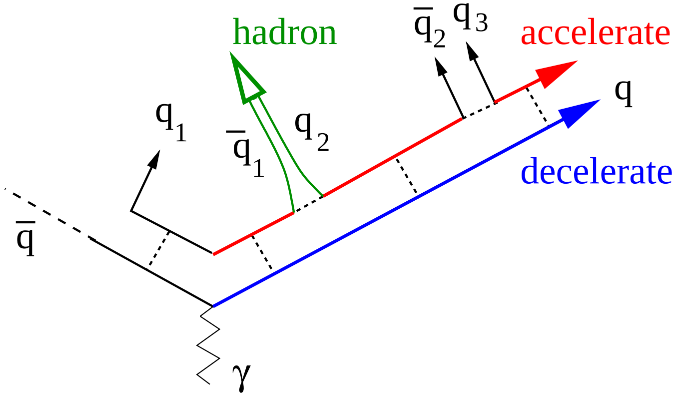
[Becattini, Castorina, Manninen, HS 2008]

Unruh temperature ∼ 1 / mass of secondary

we had for finite quark mass m_q

$$a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}} \quad \Rightarrow \quad T_U = \frac{a_q}{2\pi}$$

produced meson consists of quarks $\bar{q}_1$ and q_2



meson containing two different quark masses
will have average acceleration

$$\bar{a}_{12} = \frac{w_1 a_1 + w_2 a_2}{w_1 + w_2} = \frac{2\sigma}{w_1 + w_2}; \quad w_i \simeq \sqrt{m_i^2 + (\sigma/2\pi)}$$

leading to

$$T(12) \simeq \frac{a_{12}}{2\pi}$$

easily extended to baryons; result: five temperatures

$T(00) = T(000);\ T(s0);\ T(ss) = T(sss);\ T(00s);\ T(0ss)$

fully determined by σ and m_s

for $\sigma \simeq 0.17$ GeV^2 and $m_s \simeq 0.08$ GeV

obtain temperatures:

T	[GeV]
$T(00)$	0.164
$T(0s)$	0.156
$T(ss)$	0.148
$T(000)$	0.164
$T(00s)$	0.158
$T(0ss)$	0.153
$T(sss)$	0.148

does this work?

analyse all high energy e^+e^- data

hadron production data in e^+e^- annhilation exist at

$$\sqrt{s} = 14,\ \ 22,\ \ 29,\ \ 35,\ \ 43, 91, 180 \text{ GeV}$$

(PETRA, PEP, LEP)

example:

long-lived hadrons produced at LEP for $\sqrt{s} = 91.25$ GeV

fit data in terms
of σ and m_s

result:

$\sigma = 0.169 \pm 0.002$ GeV^2

$m_s = 0.083$ GeV

$\chi^2/\text{dof} = 23/12$

standard values:

$\sigma = 0.195 \pm 0.030$ GeV^2

$m_s = 0.095 \pm 0.025$ GeV

$e^+e^- \ \sqrt{s} = 91.2\ GeV$

species	measured			fit
π^+	8.50	±	0.10	8.30
π^0	9.61	±	0.29	9.67
K^+	1.127	±	0.026	1.089
K^0	1.038	±	0.001	1.049
η	1.059	±	0.996	0.910
ω	1.024	±	0.059	0.971
p	0.519	±	0.018	0.557
η'	0.166	±	0.047	0.096
ϕ	0.0977	±	0.0058	0.1060
Λ	0.1943	±	0.0038	0.1891
Σ^+	0.0535	±	0.0052	0.0437
Σ^0	0.0389	±	0.0041	0.0444
Σ^-	0.0410	±	0.0037	0.0400
Ξ^-	0.01319	±	0.0005	0.01269
Ω	0.00062	±	0.0001	0.00077

illustration:
ϕ production in H-U vs. standard statistical model

ϕ production density in standard statistical model

$$\langle n \rangle_\phi = 3\frac{Tm^2}{2\pi^2}\mathrm{K}_2(m/T)\ \gamma_S^2$$

with $T \simeq 165$ MeV, $\gamma_S \simeq 0.65$: $\langle n \rangle_\phi \simeq 1.85\ \gamma_S^2 \simeq 0.078$

NB: $\gamma_S^2 \simeq 0.42$ reduces equilibrium rate by more than 2

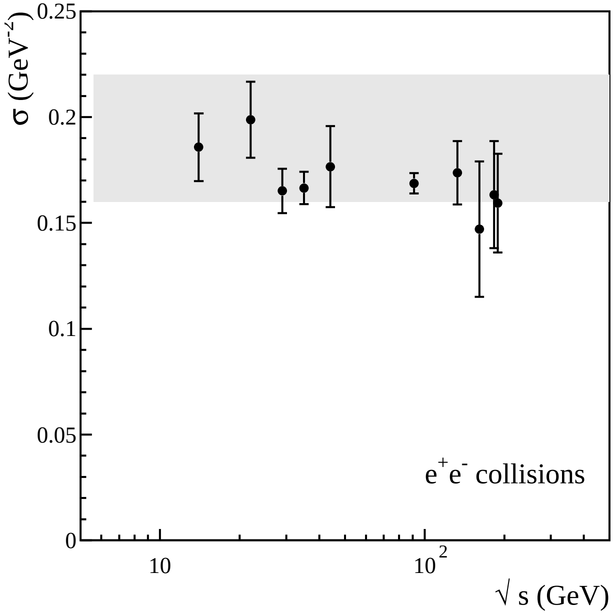
ϕ production density in H-U statistical model

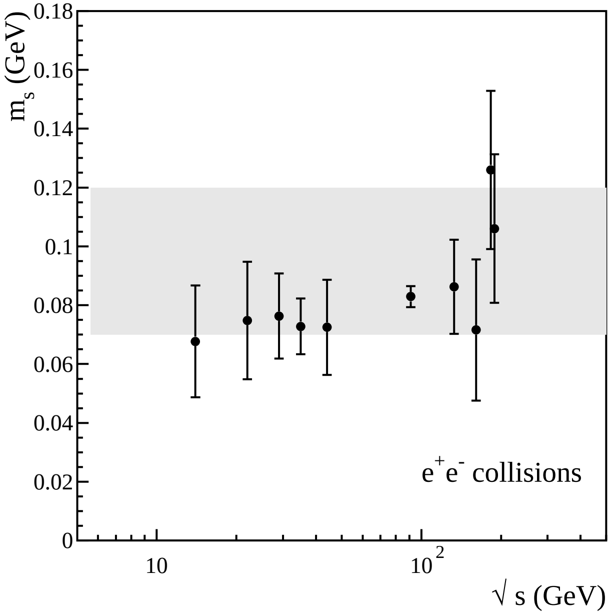
$$\langle n \rangle_\phi = 3\frac{T(ss)m^2}{2\pi^2}\mathrm{K}_2(m/T(ss))$$

with $T(ss) \simeq 148$ ($vs.$ 164) MeV: $\langle n \rangle_\phi \simeq 0.077$

[NB: actual production rates ∼ heavy flavor decay]

results from all data





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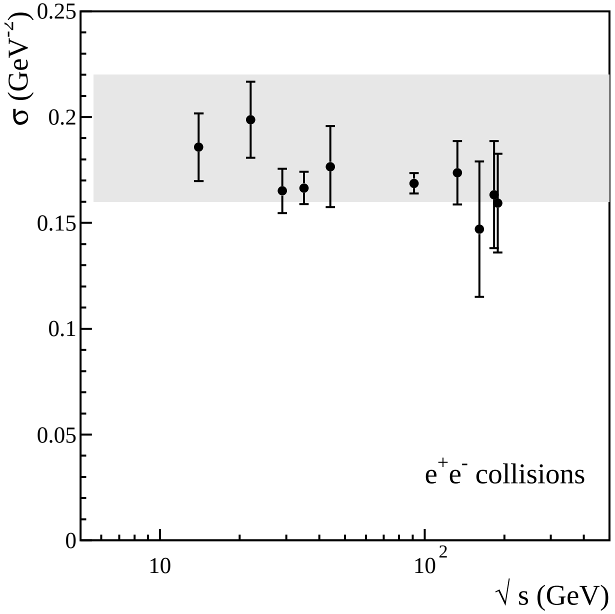
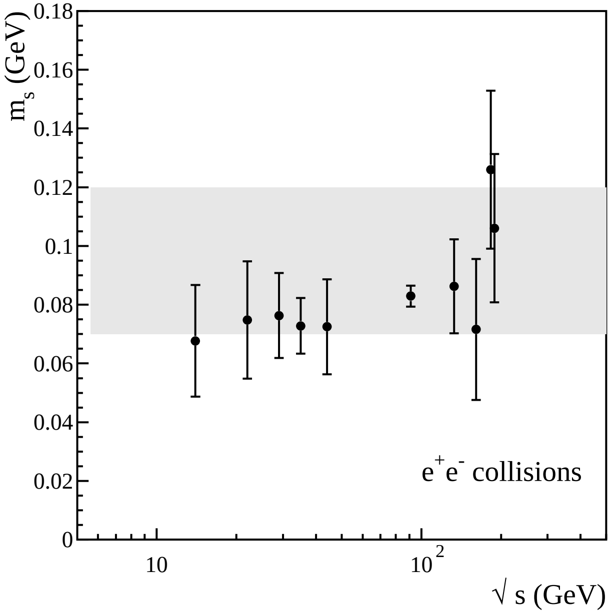
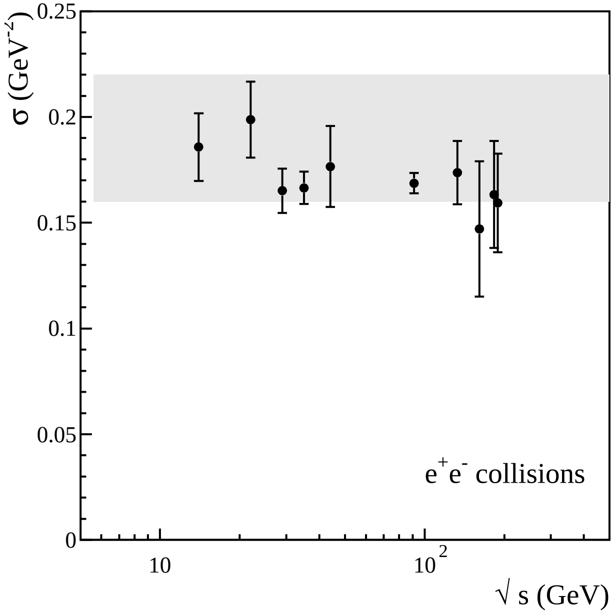


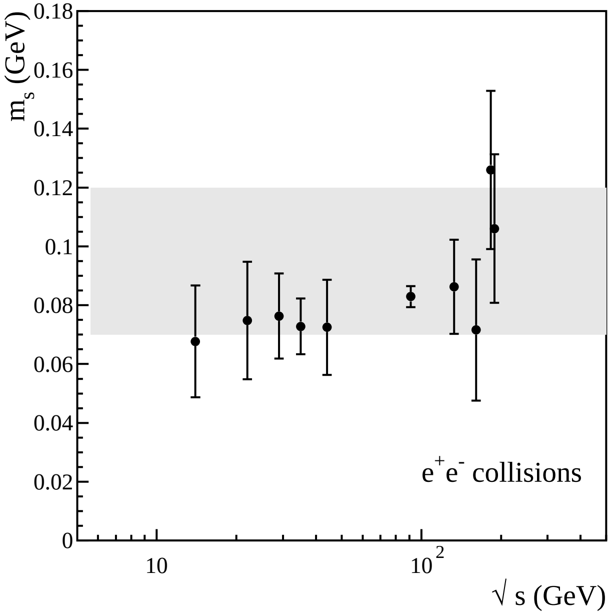


Conclude

thermal hadron production in e^+e^- annihilation, includ'g strangeness suppression, is reproduced parameter-free as Hawking-Unruh radiation of QCD

results from all data





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thermal hadron production in e^+e^- annihilation, includ'g strangeness suppression, is reproduced parameter-free as

Hawking-Unruh radiation of QCD

$\Rightarrow$ $pp/p\bar{p}$ (straight-forward); heavy ions (interesting)

Heavy Ions

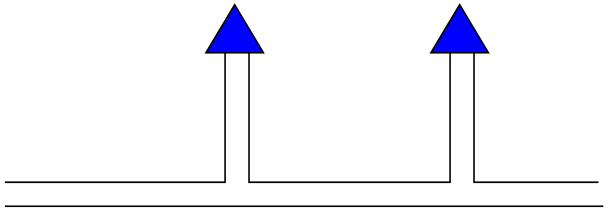
- elementary collisions

  sequential $q\bar{q}$ pair production ⇒ independent hadron emission

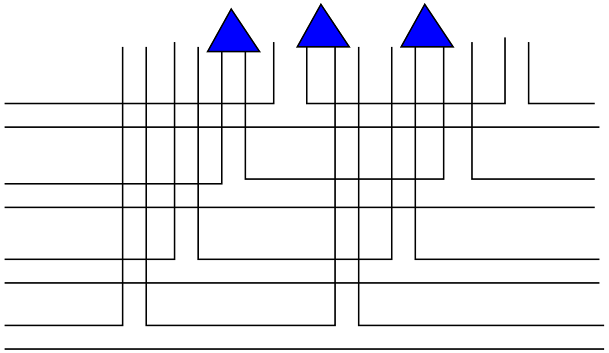
- nuclear collisions

  superposition of $q\bar{q}$ pair production, interference

  exogamous pairing, not hadronic scattering



elementary



nuclear

result: increase in strange hadron temperatures

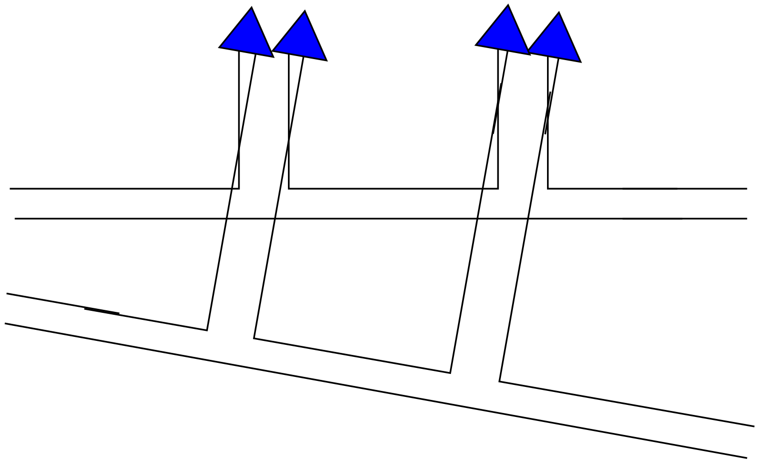
$$T(0s) \;\to\; [T(00)+T(0s)]/2 \equiv T_r(0s) > T(0s)$$

$$T(ss) \;\to\; [T(0s)+T(0s)]/2 \equiv T_r(ss) > T(ss)$$

T_r			T
$T_r(00)$	$T(00)$	0.164	0.164
$T_r(0s)$	$[T(00)+T(0,s)]/2$	0.160	0.156
$T_r(ss)$	$T(0s)$	0.156	0.148
$T_r(00s)$	$[2\;T(00)+T(0s)]/3$	0.161	0.158
$T_r(0ss)$	$[T(00)+2\;T(0s)]/3$	0.159	0.153
$T_r(sss)$	$T(0s)$	0.156	0.148

corresponds to $\gamma_s \simeq 0.82$ (vs. 0.65)
strangeness suppression is considerably reduced

Further nuclear effect:
transverse momentum broadening



- initial state collisions → rotation of emission axes
- quarks from different NN collisions not collinear
- exogamous pairing broadens p_T distribution
- NB: combination of initial & final state effects

5. Kinetic vs. Stochastic Thermalization

Kinetic thermalization:

time evolution of given non-equilibrium configuration
(two parallel colliding parton beams)
through multiple collisions
to a time-independent equilibrium state
(quark-gluon plasma)
requires

- **many constituents**
- **sufficiently large interaction cross sections**
- **sufficiently long time**

thermal hadron production in e^+e^-, $pp/p\bar{p}$?

Hagedorn: *the emitted hadrons are "born into equilibrium"*

Hawking-Unruh radiation:

- final state produced at random from the set of all states corresponding to temperature T_H determined by confining field

- this set of all final states is same as that produced by kinetic thermalization

- measurements cannot tell if the equilibrium was reached by thermal evolution or by throwing dice:

$\Rightarrow$ Ergodic Equivalence Principle $\Leftarrow$

gravitation $\sim$ acceleration kinetic $\sim$ stochastic

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- Nuclear collisions: exogamous pairing reduces strangeness suppression, causes p_T broadening.
- Given string tension σ and strange quark mass m_s, obtain parameter-free description of thermal hadron production in high energy interactions.

God does play dice, but He sometimes throws them where they can’t be seen.

Stephen Hawking